2017 Physics

1

1 Estimate the de Broglie wavelength of a tennis ball moving at 10 ms^{-1} and explain why the ball does not exhibit wavelike properties in practical experiments. [5]

2

2 A resistance R is in series with a capacitor C. At angular frequency ω the magnitude of the complex impedance Z of this combination is given by

$$|Z|^2 = |R|^2 + \frac{1}{|\omega C|^2}.$$

Find $|Z|$ and the error in $|Z|$, given that $R = 100\ \Omega \pm 2\%$, $\omega = 1000 \pm 10$ rad s^{-1}, and $C = 5.00 \pm 0.05\ \mu\mathrm{F}$. [5]

3

3 A webcam has a lens of focal length 4 mm. It is used to image a 50 cm object which is located at 30 cm from the lens and is in focus at the camera's sensor. Calculate the size of this image. [5]

4

4 Consider two identical pendulum clocks. One is placed at the bottom of a 1 km deep mine shaft while the other one stays at the earth's surface. The clocks are synchronized at $t = 0$. After one hour, calculate the time difference between the clocks. You may assume the earth to be a sphere with a radius 6371 km and to have a uniform density. [5]

5

5 The acoustic resonance frequency f (s^{-1}) of an open, gas-filled tube depends only on the molar gas constant R ($\mathrm{J\,K^{-1}\,mol^{-1}}$), the absolute temperature T (K), the molecular mass of the gas M ($\mathrm{kg\,mol^{-1}}$) and the length of the tube L. Given that

$$f \propto R^{\alpha} T^{\beta} M^{\gamma} L^{\delta}$$

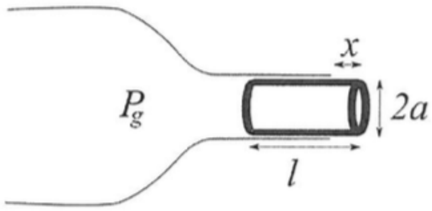
use dimensional analysis to find α, β, γ, and δ. [5]

6

6 A sinusoidal voltage of angular frequency ω is applied across an inductor L and resistance R connected in series. Find the amplitude and phase of the voltage across the resistor relative to the applied voltage when $R = \omega L$. [5]

7

7 A cylindrical cork, with density ρ, length l and radius a, protrudes a distance x from the neck of a bottle containing a gas at pressure P_g above atmospheric pressure, as shown below.



The portion of the cork in the bottle is compressed, and pushes radially outwards on the bottle neck with a constant pressure P_c. Show that the cork will slide outwards if

$$P_g > \frac{2(l-x)\mu}{a} P_c,$$ [3]

where μ is the coefficient of friction between the cork and the glass. Assume μ takes the same value whether or not the cork is slipping.

The cork starts flush with the bottle ($x = 0$), but the pressure in the bottle is just enough for the cork to start slipping forwards. Assuming the pressure in the bottle is constant, and the cork starts with a small outward velocity v_0 at $t = 0$, find the equation of motion for the cork and, by substitution or otherwise, show the motion of the cork is

$$x = c \sinh(bt)$$

and give expressions for c and b in terms of P_g, ρ, l and v_0. [5]

Show that, if v_0 is negligibly small, the cork exits the bottle with velocity

$$v = \sqrt{\frac{P_g}{\rho}}.$$ [4]

If the density of cork is 0.5 g cm^{-3}, and a typical champagne bottle is pressurized to 400 kPa above atmospheric pressure, find (neglecting air resistance) to what height it is possible to fire a cork. [2]

An actual champagne cork does indeed start with a length l within the bottle, but also has a stopper on the end that doubles the mass of the cork. How will this modification change the maximum achievable height? [1]

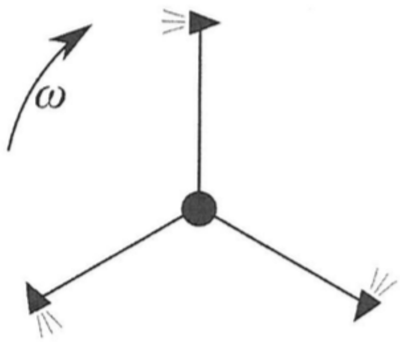
8

8 Define the total momentum and the total angular momentum (around the origin) of an isolated set of N interacting particles in which the ith particle has mass m_i, position $\boldsymbol{r}_i$ and velocitiy $\boldsymbol{v}_i$, and discuss under what circumstances these quantities are conserved. [3]

An isolated rocket with empty mass m carries fuel of mass m_f, and propels itself forwards by ejecting fuel backwards through its thruster at a velocity v_e relative to the rocket. Using conservation of momentum, show that if the rocket starts at rest then, when it has ejected all its fuel, it is moving at a velocity

$$v = v_e \ln\left(\frac{m + m_f}{m}\right).$$ [3]

Three such rockets are mounted on rigid rods of length a and mass m (equal to the empty mass of the rocket) which are then used as the arms of a rotating firework as shown below.



Treating the rockets as point masses, find the moment of inertia of the firework when the rockets are fully fueled. [3]

Show that, if the rockets are ignited when the system is at rest, then, when the rockets have finished, the firework will be spinning with angular velocity

$$\omega_f = \frac{v_e}{a} \ln(\beta),$$

and give an expression for β. You may neglect effects from friction and air resistance. [4]

After the rockets have finished, and the firework is spinning at ω_f, it becomes subject to a constant frictional torque G from its axle. Find the further angle the firework will turn through before it comes to rest. [2]

[*A uniform rod, with mass m and length l, has a moment of inertia* $I = \frac{1}{12} m l^2$ *about a perpendicular axis through its centre of mass.*]

9 A train is travelling away from a station at a constant relativistic speed v in the positive x-direction in the station's frame S. It is equipped with a pulsed radio beacon that emits pulses at frequency f_0 in the train's frame S'. Pulses are emitted from the back of the train to an observer standing on the platform. Using a diagram or otherwise, find an expression for the frequency f at which the pulses are detected by this observer. [3]

Show that for small velocities $v \ll c$, this result agrees with the non-relativistic Doppler effect $f/f_0 = (1 - v/c)$. [2]

Two identical lights are now attached to the front and the back of a train, travelling at the same constant relativistic speed v. As the train approaches the station, its headlight appears green ($\lambda_1 = 530$ nm) and the tail light, as it departs, appears red ($\lambda_2 = 636$ nm). Calculate the value of v in terms of c. [3]

What is the wavelength of the lights measured when the train is at rest in the station? [3]

A blue light, emitting at a wavelength $\lambda'_3 = 400$ nm in its rest frame, is now mounted to the side of a train traveling at a constant speed of $v = 0.6c$ in the station's frame. Calculate the wavelength of the light measured by an observer at the platform at the exact moment at which the train passes. You may neglect the distance between the observer and the train. [4]

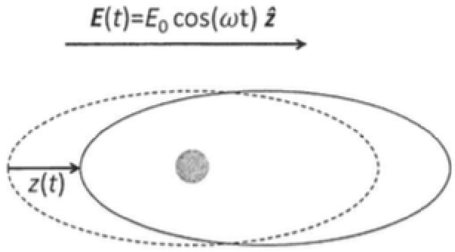
[*The Lorentz transformations between frames S and S′, where S′ is moving at a velocity v relative to S in the x-direction are*

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2).$$
]

10

10 An electromagnetic wave from the Sun, with electric field $\boldsymbol{E}(t) = E_0 \cos(\omega t)\hat{z}$, is incident on a particle in the atmosphere. To understand the interaction between the field and the particle, we model the particle as a heavy stationary core surrounded by a rigid cloud of charge q and mass m. The electric field induces a displacement z of the cloud from its equilibrium position centred on the core as shown below.



For small z, explain why we expect the potential energy of the cloud to be proportional to z^2, and why the cloud feels a restoring force back to its equilibrium position proportional to z. [2]

If this restoring force is $-kz$, and the cloud also feels a damping force $-b\dot{z}$, show that its equation of motion is of the form

$$\ddot{z} + 2\gamma\dot{z} + \omega_0^2 z = \frac{q}{m}E(t),$$

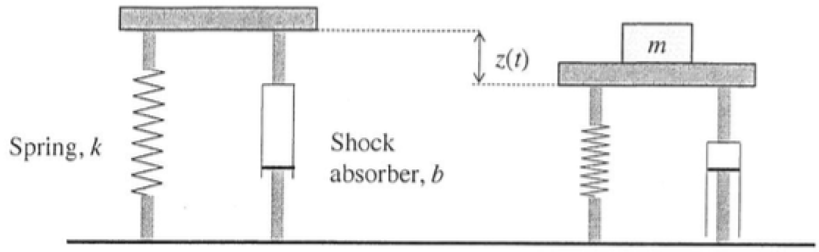
and give expressions for ω_0 and γ. [2]

Using a complex representation of the steady-state motion $z = z_0 \exp(\mathrm{i}(\omega t + \phi))$, find the amplitude z_0 of the motion of the cloud, and the phase-shift ϕ between the motion of the cloud and the electromagnetic wave. [6]

The cloud is made of electrons so that its charge is $q = -ne$ and its mass is $m = nm_e$. The electrons in the cloud emit radiation with intensity proportional to the square of the amplitude of their acceleration. Given that $k/n = 100\,\mathrm{kg\,s^{-2}}$ and that visible radiation lies between wavelengths 400 nm and 800 nm, explain, by considering the values of ω and ω_0, why the sky is blue. [5]

11

11 A weighing scale is modelled as a plate resting on a spring and shock absorber in parallel. At time $t = 0$, a mass m is placed on the scale and the plate then moves vertically by $z(t)$ as shown below. The mass of the scale is negligible compared to m, the spring has constant k, and the shock absorber produces a force $-b\dot{z}$.



Write down the total energy of the system and determine the height difference between the equilibrium positions of the unloaded and the loaded scale. [2]

Explain why the system loses energy at a rate $\mathrm{d}E/\mathrm{d}t = -b\dot{z}^2$, and hence deduce the equation of motion of the mass. [2]

Sketch the position of the mass $z(t)$ as a function of time $t > 0$, when the system is lightly, critically and heavily damped. [3]

Explain why an instrument maker would design the system to be critically damped, and find the corresponding expression for b. [2]

At time $t = 0$, the mass is placed on the scale with an initial velocity v_0. In the case of critical damping, show that the motion is of the form

$$z(t) = (A + B\,t)\,\exp(-\gamma\,t) + C,$$

and determine the coefficients A, B and C. [3]

How large does v_0 need to be for the scales's reading to overshoot before it settles? [3]

12

12 The quantum-mechanical wavefunction, $\psi(x, y)$, describing a particle of mass m in two dimensions satisfies the Schrödinger equation

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+V(x, y)\psi = E\psi .$$

Explain the meaning of the symbols $V(x, y)$ and E, and discuss briefly the significance of this equation and the physical interpretation of the wavefunction $\psi(x, y)$. [4]

A particle is confined to a two-dimensional box having dimensions $0 \le x \le a$ and $0 \le y \le b$. This box can be modelled as an infinitely deep two-dimensional potential well, with $V = 0$ in the interior of the box, and $V = \infty$ elsewhere. Show that the wave function satisfying the Schrödinger equation within this potential well has the general form

$$\psi(x, y) = A\sin(k_x x)\sin(k_y y) .$$

Find the permitted values of (k_x, k_y), and hence derive an expression for the possible energy values of the system. [6]

A photodetection device has been built that traps an electron in a 2D rectangular potential well as described above, with $a = 1$ nm and $b = 2$ nm. A photon of energy ϵ_ϕ can be absorbed by exciting the electron from its ground state, of energy E_i, to an excited state of energy E_f, such that $\epsilon_\phi = E_f - E_i$. Calculate the lowest photon energy that can be absorbed, expressing the answer in eV. [2]

For $a = 1$ nm, obtain a new value of b (larger than a) such that neither of the lowest two energy states are degenerate but the third energy level is doubly degenerate. [3]

13

13 The transverse displacement y, for sinusoidal waves travelling in the x-direction on a string under tension T, is given by

$$y = y_0\cos(\omega t - kx),$$

where ω and k are both positive. Sketch the displacement pattern at time $t = 0$ and at time $t = \pi/2\omega$. [2]

Explain why this describes the motion of a wave travelling in the positive x direction, and show that the velocity of the wave c is equal to ω/k. [3]

Derive the wave equation for a string of mass density ρ under tension T by applying Newton's second law to a small segment of the string and using a carefully labelled diagram. Show that the wave velocity $c = \sqrt{T/\rho}$. [3]

For a string of $\rho = 20$ g m^{-1} mass density under a tension $T = 1000$ N compute the frequency of a wave of wavelength of 0.5 m. [3]

Consider a pulse-like wave in a string under tension such that at time $t = 0$ the transverse displacement is

$$y(x, 0) = Ae^{-ax^2/2}$$

and the particle velocity is

$$\left.\frac{\partial y}{\partial t}\right|_{t=0} = c\,x\,aAe^{-ax^2/2}.$$

Write down the wave function $y(x, t)$ and compare this to the situation with the same initial transverse displacement but initial particle velocity equal to zero everywhere. [4]

14

14 State Gauss' Law in electrostatics. [2]

Consider a sphere of uniform positive charge density ρ_0 and radius R. Calculate the radial electric field E and the radial electric potential V, as functions of radial distance r from the centre of the sphere. Sketch graphs of the radial electric field E and the radial electric potential V as a function of r. [5]

A hydrogen atom in its ground state has an electron charge density given by

$$\rho(r) = -\frac{e}{\pi a_0^3} \exp(-2r/a_0),$$

where a_0 is the Bohr radius. Show that the radial electric field due to the electron cloud is given by

$$E(r) = \frac{e}{4\pi\epsilon_0}\left(\frac{(e^{-2r/a_0} - 1)}{r^2} + \frac{2e^{-2r/a_0}}{a_0 r} + \frac{2e^{-2r/a_0}}{a_0^2}\right).$$

[4]

A point charge of the same magnitude as the charge of the nucleus of the hydrogen atom is placed at a distance $r \gg a_0$ away from the atom. Explain why a force arises on the neutral hydrogen atom, and discuss how the forces change when the sign of the charge changes. [4]

15

15 Write short notes on two of the following topics:

(a) the motion of charged particles in electric and magnetic fields; [7½]

(b) conservative fields and potential energy; [7½]

(c) the Biot-Savart Law and its uses; [7½]

(d) Maxwell's equations. [7½]