2012 Mathematics (1)

Section A

1

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6

Express $\dfrac{4x^2 - x}{(x-1)^2(x+2)}$ as partial fractions. [2]

Solution(s):

From user: lester

2012 Paper 1. IA MNST. 6.

$$\frac{4x^2-x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \qquad (*)$$

"$\times$ by $(x+2)$" and let $x \to -2$. get $\dfrac{4.4+2}{(-3)^2} = C \Rightarrow C = 2$.

"$\times$ $(x-1)$" $\Rightarrow$ $\dfrac{4x^2-x}{(x-1)(x+2)} = A + \dfrac{B}{x-1} + \dfrac{C(x-1)}{x+2}$.

Let $x \to \infty$. Then $4 = A + 0 + C \Rightarrow A = 4 - C = 4 - 2 = 2$

Finally "$\times$ $(x-1)^2$" $\Rightarrow$

$$\frac{4x^2-x}{x+2} = A(x-1) + B + \frac{C(x-1)^2}{x+2}$$

and let $x \to 1$ giving $\dfrac{4-1}{3} = 0 + B + 0 \Rightarrow B = \cancel{3}\,1$

So: $(*) = \dfrac{2}{x-1} + \dfrac{1}{(x-1)^2} + \dfrac{2}{x+2}$.

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Section B

11X

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12Z

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Solution(s):

From user: ar857

2012 Paper 1 Q 12Z

12Z

a) i) $\left(\frac{2+i}{-1+i}\right)^2 = \left(\frac{2+i}{-1+i}\cdot\frac{-1-i}{-1-i}\right)^2 = \left(\frac{-2-i-2i-i^2}{2}\right)^2 = \frac{1}{4}(-1-3i)^2$

$= \frac{1}{4}(1-9+6i) = -2+\frac{3}{2}i$

ii) $(1+i)^{10} = \left(\sqrt{2}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\right)^{10} = 32(0+i) = 32i$

iii) $\sin(\frac{\pi}{2}+i\ln 2) = \sin\frac{\pi}{2}\cos i\ln 2+\cos\frac{\pi}{2}\sin i\ln 2$

$= \cos(i\ln 2) =$

$= \cosh(\ln 2) = \frac{2+\frac{1}{2}}{2} = \frac{5}{4}$

b) $z^2 = (i-1) = \sqrt{2}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right) = \sqrt{2}\cos\left(\frac{3}{4}\pi+2\pi\right)+i\sin\left(\frac{3}{4}\pi+2\pi\right)$

$z = \sqrt[4]{2}\left[\cos\left(\frac{3}{8}\pi+n\pi\right)+i\sin\left(\frac{3}{8}\pi+n\pi\right)\right]$

c) $\sin^5\phi = \frac{1}{32(i)^5}\cdot(e^{x}-e^{-x})^5$ $\quad \phi = x$

$= \frac{1}{16}\cdot\frac{1}{2i}\left(e^{5x}-5e^{4x}e^{-x}+10e^{3x}e^{-2x}-10e^{2x}e^{-3x}+5e^{x}e^{-4x}-e^{-5x}\right)$

$= \frac{1}{16}\left(\sin 5\phi-5(\sin 3\phi)+10\sin\phi\right)$

d) $\cosh z = -1$

$\frac{A+\frac{1}{A}}{2} = -1 \qquad A+\frac{1}{A} = -2 \qquad A^2+2A+1=0 \qquad e^{z} = -1 \qquad z = \ln(-1) = \ln\left(e^{(\pi+2\pi n)i}\right)$

2012 Paper I Q13

13Z

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Solution(s):

From user: ar857

2012 Paper I Q13

$A^2+2A+1=0$

$z = \ln(-1) = \ln(e^{(\pi+2\pi n)i})$

$z = \pi i(1+2n)$

13) $\frac{dy}{dx} = \frac{y^2+xy}{x^2} = \frac{y^2}{x^2} + \frac{y}{x}$

SUBSTITUTION

$y = vx$ $\quad v = \frac{y}{x}$

$\frac{dy}{dx} = \frac{dv}{dx}x + v$

a) $\frac{dv}{dx}x + v = v^2 + v$

$dv \frac{1}{v^2} = \frac{1}{x}dx$

$-\frac{1}{v} = \ln x + c$

$-\frac{x}{y} = \ln x + c$ $\quad y = \frac{-x}{\ln x + c}$

b) $\frac{d(x+y)}{dy} = 1 = \frac{d(x)}{dx} = 1$ $\Rightarrow$ exact differential

$x\frac{dy}{dx} + x + y = 0$

general solution $\frac{1}{2}x^2 + yx = c$ $\quad y = \frac{(c - \frac{1}{2}x^2)}{x}$

From user: ar857

c)

$\frac{dy}{dx} + ky = a\sin mx$

$\int e^{\int k\,dx} = e^{kx}$

$e^{kx}y = \int a\sin mx\, e^{kx}$

$= a\int \sin mx\, e^{kx} = (\sin mx \frac{1}{k}e^{kx} - \int \frac{m}{k}e^{kx}\cos mx) a$

$= aI_{m,k} = a(\sin mx \frac{1}{k}e^{kx} - \frac{m}{k}(\frac{1}{k}e^{kx}\cos mx + \int \frac{m}{k}e^{kx}\sin mx))$

$= aI_{m,k} = a(\sin mx \frac{1}{k}e^{kx} - \frac{m}{k^2}e^{kx}\cos mx - \frac{m^2}{k^2}I_{mk})$

$aI_{mk} = \frac{a(\sin mx \frac{1}{k}e^{kx} - \frac{m}{k^2}e^{kx}\cos mx)}{1+\frac{m^2}{k^2}} + c$

$1 = \frac{-a\frac{m}{k^2}\cdot k^2}{k^2+m^2} + c$

$\therefore \frac{k^2+m^2+am}{k^2+m^2} = c$

$y = \frac{a(k\sin mx - m\cos mx) + (k^2+m^2+am)e^{-kx}}{k^2+m^2}$

14S

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15T

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Solution(s):

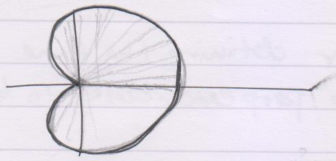
From user: ar857

2012 Paper I Q15T

(15) $r = 1+\cos\phi$ $\quad x = r\cos\phi$

$y = r\sin\phi$

a)



$y = r\sin\phi$

$y = \sin\phi + \sin\phi\cos\phi$

$\frac{dy}{d\phi} = \cos\phi + \cos\phi^2 - \sin\phi^2$

$2\cos\phi^2 + \cos\phi - 1 = 0$

$(2\cos\phi - 1)(\cos\phi + 1)$

$\cos\phi = -1$ minimum

$\cos\phi = \frac{1}{2}$ maximum

$y = (1+\frac{1}{2})\cdot\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$

$x = (1+\frac{1}{2})\cdot\frac{1}{2} = \frac{3}{4}$

maximum $[\frac{3}{4}, \frac{3\sqrt{3}}{4}]$

b) $\int_0^{2\pi}\int_0^{1+\cos\phi} r\,dr\,d\phi$

$\int_0^{2\pi} \frac{(1+\cos^2\phi+2\cos\phi)}{2}\,d\phi = \int_0^{2\pi}\left(\frac{1}{2}+\frac{1}{4}+\frac{\cos 2\phi}{2}+\cos\phi\right)d\phi$

$= \pi + \pi/2 + 0 \quad 3\pi/2$

c) $(dx)^2 = (dr\cos\phi)^2 - 2dr\cos\phi\sin\phi\,d\phi + r^2\sin^2\phi\,(d\phi)^2$

$(dy)^2 = (dr\sin\phi)^2 + 2dr\sin\phi\cos\phi\,d\phi + r^2\cos^2\phi\,(d\phi)^2$

$dx^2+dy^2 = (dr)^2 + r^2(d\phi)^2$

$\int_T \sqrt{(dx)^2+(dy)^2} = \int_0^{2\pi}\sqrt{\left(\frac{dr}{d\phi}\right)^2 + \left(\frac{r\,d\phi}{d\phi}\right)^2}\,d\phi$

$\sqrt{2}\int_0^{2\pi}\sqrt{1+\cos\phi} = \sqrt{2}\int_0^{2\pi}\sqrt{2\cos^2(\frac{\phi}{2})}$

$= 2\int_0^{2\pi}\sqrt{2\cos^2\frac{\phi}{2}} = 4\int_0^{\pi}\cos\frac{\phi}{2} = 8\cdot 1 = 8$

16Y

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Solution(s):

From user: ar857

(16) 2012 Paper 1 Q16 Y

a) $\nabla f = \frac{-2}{(1+x^2+y^2)^2}(x,y)$

$\nabla f(1,0)\cdot\frac{v}{|v|} = \frac{-2}{(2)^2}(1,0)\cdot\left(\frac{4}{5},\frac{3}{5}\right)$

$= \frac{-2}{4}\cdot\frac{4}{5} = -\frac{2}{5}$

b)

$\frac{\partial g}{\partial x} = \left(2x - x(x^2-y^2)\right)\cdot e^{-\frac{x^2+y^2}{2}}$

$= x\cdot(2-x^2+y^2)\, e^{-\frac{x^2+y^2}{2}}$

$= 0$ for $x=0$ or $x^2=2+y^2$

$\frac{\partial g}{\partial y} = (-2-x^2+y^2)\, y\, e^{-\frac{x^2+y^2}{2}}$

$= 0$ for $y=0$ or $y^2=2+x^2$

Stationary points at

[0, 0]

[0, ±√2]

[±√2, 0]

c) $\frac{\partial^2 g}{\partial x^2} = (2-x^2+y^2)e^{-\frac{x^2+y^2}{2}} - 2x^2 e^{-\left(\frac{x^2+y^2}{2}\right)} - 2x^2(2-x^2+y^2)e^{-\frac{x^2+y^2}{2}}$

$g_{xx}(0,0) = 2 - 0 - 0 = 2$

$g_{yy} = -2$

since g_{yy} has differere sign tha g_{xx} ⇒ Saddlepoint

17R

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18R

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Solution(s):

From user: ar857

(18) 2012 Paper I Q 18 R

Ben goes $\frac{3}{4}$

Ben stays $\frac{1}{4}$

a) i) $\left(\frac{3}{4}\right)^{24}$

ii) $\left(\frac{1}{4}\right)^{24}$

iii) $\frac{3}{4} \cdot 24 = 18$

iv) $\sigma^2 = \frac{3}{4} \cdot \frac{1}{4} \cdot 24 = \frac{9}{2}$

$\sigma = \frac{3}{\sqrt{2}}$

b) i) $\int_\alpha^\beta f(x)\,dx$

ii) $\mu = \langle x \rangle = \int_\alpha^\beta x f(x)\,dx$

$\sigma^2 = \int_\alpha^\beta (x-\mu)^2 f(x)\,dx = \int_\alpha^\beta x^2 f(x)\,d(x) - (\mu)^2$

From user: ar857

c) $Axe^{-\lambda x}$ for $\lambda \geq 0$

i) $\int_0^\infty Axe^{-\lambda x} = A\frac{1}{\lambda}\int_0^\infty e^{-\lambda x} = A\frac{1}{\lambda^2} = 1 \implies \underline{A = \lambda^2}$

ii) $\int_0^\infty Ax^2e^{-\lambda x} = \frac{2}{\lambda}\int_0^\infty Axe^{-\lambda x} = \frac{2}{\lambda} = \mu$

$\sigma^2 = \int_0^\infty Ax^3e^{-\lambda x} = \int_0^\infty \left(\frac{1}{\lambda} A e^{-\lambda x} \cdot 3x^2\right)dx - \frac{4}{\lambda^2}$

$= \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2} = \sigma^2$

iii) $\frac{2}{\lambda} + 2\frac{\sqrt{2}}{\lambda} = \frac{2+2\sqrt{2}}{\lambda}$

$p(X > \mu + 2\sigma) = \int_{\frac{2+2\sqrt{2}}{\lambda}}^\infty \lambda^2 x e^{-\lambda x}$

$= \lambda^2 \left[-\frac{1}{\lambda} x e^{-\lambda x}\right]_{\frac{2+2\sqrt{2}}{\lambda}}^\infty + \frac{\lambda^2}{\lambda}\int_{\frac{2+2\sqrt{2}}{\lambda}}^\infty e^{-\lambda x}\,dx$

$= \frac{\lambda^2}{\lambda}\frac{2+2\sqrt{2}}{\lambda} e^{-2-2\sqrt{2}} + \frac{\lambda^2}{\lambda^2} e^{-2-2\sqrt{2}}$

$= \left(3 + 2\sqrt{2}\right) e^{-2(1+\sqrt{2})}$

19W*

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20Y*

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