2011 Mathematics (1)

Section A

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Section B

11S

(a) Write down the first four terms of the Taylor expansion of a function $f(x)$ about $x = a$. [4]

(b) Find, by any method, the Taylor expansion about $x = 0$, up to and including the term in x^3, of the following functions:

(i)
$$\frac{1}{(x^2+9)^{1/2}},$$
[6]

(ii)
$$\ln[(2+x)^3],$$
[4]

(iii)
$$e^{\sin x}.$$
[6]

Solution(s):

From user: lester

(a) $f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a)$
$+ O\left((x-a)^4\right).$

(b)

(i) $\frac{1}{(x^2+9)^{\frac12}} = \frac13\left(1+\left(\frac{x}{3}\right)^2\right)^{-\frac12}$

$= \frac13\left(1 - \frac12\left(\frac{x}{3}\right)^2 + O(x^4)\right)$

$= \frac13 - \frac{1}{54}x^2 + O(x^4).$

(ii) $\ln\left((2+x)^3\right) = 3\ln(2+x) = 3\ln 2 + 3\ln\left(1+\frac{x}{2}\right)$

$= 3\ln 2 + 3\left\{\frac{x}{2} - \frac{\left(\frac{x}{2}\right)^2}{2} + \frac{\left(\frac{x}{2}\right)^3}{3} + O(x^4)\right\}$

$= 3\ln 2 + \frac32 x - \frac38 x^2 + \frac18 x^3 + O(x^4).$

(iii) $e^{\sin x} = e^{x - \frac{x^3}{3!} + O(x^5)}$

$= 1 + \left(x - \frac{x^3}{3!} + O(x^5)\right) + \frac{1}{2!}\left(x^2 + O(x^4)\right) + \frac{1}{3!}\left(x^3 + O(x^5)\right) + O(x^4)$

$= 1 + x + \frac12 x^2 + O(x^4).$

12X

The Dean of Porterhouse (the oldest and most famous of the colleges of the University of Carrbridge) leads a service in Chapel every Sunday. The length s of each service, in minutes, is exponentially distributed with a mean of $\bar{s}$ minutes.

(a) Write down the probability density function for s.

Unfortunately, some of the Dean's services are being interrupted as a result of an electrical fault in the chapel organ. This fault causes one of the organ's pipes to spontaneously emit a loud sound t minutes after the beginning of the service. It is found that t is exponentially distributed with mean $\bar{t}$ minutes and is independent of s.

(b) Draw a pair of axes at right-angles to each other labelling one s and one t. Indicate on this diagram the region of the (s,t)-plane in which the service is **not** interrupted by the organ.

(c) The probability of being in some region of this plane is the double integral of the product of the density functions for s and t integrated over the region. Explain in words why this is so.

(d) Calculate the *probability* (as a function of $\bar{s}$ and $\bar{t}$) that the service is **not** interrupted by the organ.

Define the random variable r to be equal to "-1" if the organ does not interrupt the service, and equal to "the number of minutes of the service which are remaining, at the moment the organ makes a noise" if the organ interrupts the service.

(e) On the same diagram as before, indicate the region of the (s,t)-plane in which $r > r_0$, where r_0 is a positive constant.

(f) Calculate the *probability* (as a function of r_0, $\bar{s}$ and $\bar{t}$) that r is greater than r_0 minutes, again assuming that r_0 is a positive constant.

(g) Consider the answer for $P(r > r_0)$ that you have found in (f), and comment on whether it seems sensible in each of the following limits:

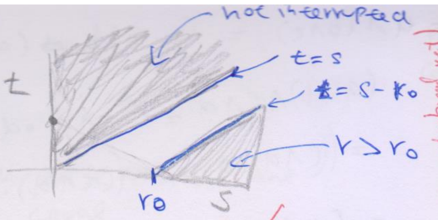
(i) $r_0 \to \infty$, (ii) $r_0 \to 0$ in the case $\bar{s} = \bar{t}$.

Suppose that it is 6:54pm, that the service was interrupted by the organ at 6:22pm, and that the Dean is *still* talking. The Fellows are getting hungry, and are wondering how many *more* minutes, m, they are going to have to stay sitting in Chapel until the service ends.

(h) State (or calculate) the expected value of m.

Solution(s):

From user: ar857



$\lambda e^{-\lambda x}$ $\frac{1}{\lambda} = \bar{s}$

a) $PDF(s) = \frac{1}{\bar{s}} e^{-\frac{1}{\bar{s}} s}$

b) Service not interrupted if $t \geq s$

c) $\iint \frac{1}{\bar{s}} e^{-\frac{s}{\bar{s}}} \frac{1}{\bar{t}} e^{-\frac{t}{\bar{t}}} \, ds \, dt$

- probability of being at any concrete point in the plane is probability of being at a concrete s times probability of being at concrete t
- if we want to find out the probability of being in the region we need to integrate over the region in order to sum the probabilities at each point in the region

d) $\int_0^{\infty} \int_s^{\infty} \frac{1}{\bar{s}} e^{-\frac{s}{\bar{s}}} \frac{1}{\bar{t}} e^{-\frac{t}{\bar{t}}} \, dt \, ds$

$= \int_0^{\infty} \frac{1}{\bar{s}} e^{-\frac{s}{\bar{s}}} \cdot \left[-e^{-\frac{t}{\bar{t}}} \right]_s^{\infty} ds$

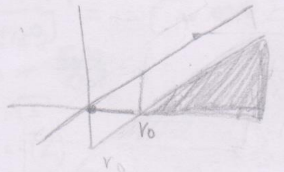
$= \int_0^{\infty} \frac{1}{\bar{s}} e^{-\frac{s}{\bar{s}}} \cdot \left[e^{-\frac{s}{\bar{t}}} \right] ds$ $e^{-s\left(\frac{\bar{t}+\bar{s}}{\bar{s}\bar{t}}\right)}$

$= \frac{1}{\bar{s}} \frac{\bar{s}\bar{t}}{\bar{t}+\bar{s}} \cdot \left[-e^{-s\frac{\bar{t}+\bar{s}}{\bar{s}\bar{t}}} \right]_0^{\infty} = \frac{\bar{t}}{\bar{t}+\bar{s}} \cdot 1 = \frac{\bar{t}}{\bar{t}+\bar{s}}$

e) $r = -1$ if organ not interrupted

$r = s - t$

$r > r_0$ only if interrupted



$\int_0^{\infty} \int_{t+r_0}^{\infty}$

$\frac{1}{\bar{t}} \int_0^{\infty} e^{-\frac{t}{\bar{t}}} \cdot \left[-e^{-\frac{s}{\bar{s}}} \right]_{t+r_0}^{\infty} = \frac{1}{\bar{t}} \int_0^{\infty} e^{-\frac{t}{\bar{t}}} e^{-\frac{t}{\bar{s}}} e^{-\frac{r_0}{\bar{s}}}$

$= \frac{1}{\bar{t}} \cdot \frac{\bar{s}\bar{t}}{\bar{s}+\bar{t}} \left[-e^{-t\cdot\left(\frac{\bar{s}+\bar{t}}{\bar{s}\bar{t}}\right) - \frac{r_0}{\bar{s}}} \right]_0^{\infty}$

$= \frac{\bar{s}}{\bar{s}+\bar{t}} e^{-\frac{r_0}{\bar{s}}}$

g) i) $r_0 \to \infty$ $P \Rightarrow 0$ yes this is to be expected, probability that $r > r_0 = \infty$ is $\Rightarrow 0$

ii) in case $\bar{s} = \bar{t}$ we would expect $P(r > r_0) = \frac{1}{2}$; $\frac{\bar{t}}{\bar{s}+\bar{t}} e^{-\frac{r_0}{\bar{s}}} = \frac{1}{2}$ if $r_0 = 0$ or $\bar{s} = \bar{t}$

h) ~~expected value of s ...~~

h) $\bar{s}$. Always $\bar{s}$.

P(stop x mins from now given not stopped yet) = re-scaled $\exp(-\frac{t}{\bar{s}})$ so still exponential

13Y

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14R

This question involves solving the differential equation

$$\sqrt{3}\frac{dy}{dx} + y = 4\sin x$$

by Fourier methods.

(a) Write down a Fourier series expansion of an arbitrary periodic function which has period 2π. [3]

(b) Suppose that $y(x)$ has such an expansion. Substitute the Fourier series expansion into the differential equation in order to obtain a constraint on its coefficients. [2]

(c) Why may we equate the coefficients of $\sin(mx)$ in this constraint (for each integer m)? You may also equate the coefficients of $\cos(mx)$. [2]

(d) By equating coefficients as described in (c), find all of the coefficients of the Fourier series expansion of $y(x)$. [5]

(e) Thus, write down the explicit form of the periodic solution $y(x)$ in only one term. [1]

(f) Sketch $y(x)$ for $0 \leqslant x \leqslant 2\pi$, clearly displaying maxima and minima. [3]

(g) Use Parseval's theorem to evaluate $\int_0^{2\pi} \{y(x)\}^2 dx$. [2]

(h) Check your answer to (g) by performing the integral explicitly. [2]

Solution(s):

From user: lester

2011 Paper 1

14R $\sqrt{3}\frac{dy}{dx} + y = 4\sin x$ ⊛

(a) $f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$ is periodic with period 2π.

↑ I don't use $a_0/2$ prefered by some others.

(b) substituting (a) into ⊛ gives

$$4\sin x = \sum_{n=1}^{\infty}\left(\sqrt{3}b_n n\cos(nx) - \sqrt{3}a_n n\sin(nx)\right) + a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$$

This tells us (equating coefficients) that:

① $a_0 = 0$

② $4 = -\sqrt{3}a_1 + b_1$ (sin x coeff)

③ $0 = \sqrt{3}b_1 + a_1$ (cos x coeff)

④ $0 = \sqrt{3}b_n n + a_n$ (cos nx coeff $n \geq 2$)

⑤ $0 = -\sqrt{3}a_n n + b_n$ (sin nx ... $n \geq 2$)

Parseval's theorem: (g).

$$\int_0^{2\pi}(y(x)^2) = b_1^2\int\sin^2 + a_1^2\int\cos^2$$
$$= b_1^2\pi + a_1^2\pi$$
$$= \pi(a_1^2 + b_1^2)$$
$$= \pi(1+3) = 4\pi$$

④ & ⑤ $\Rightarrow a_n = b_n = 0$ for $n \geq 2$ $\left(\text{since } \begin{vmatrix} 1 & \sqrt{3}n \\ -\sqrt{3}n & 1 \end{vmatrix} = 1 + 3n^2 \neq 0\right)$

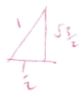
② & ③ $\Rightarrow a_1 = -\sqrt{3}b_1 \Rightarrow 4 = +3b_1 + b_1 = 4b_1 \Rightarrow b_1 = 1 \Rightarrow a_1 = -\sqrt{3}$

So: $b_1 = 1$, $a_1 = -\sqrt{3}$, all other coeffs $= 0$

($\Rightarrow f(x) = \sin x - \sqrt{3}\cos x$ which can be confirmed (check) to satisfy ⊛)

(c) Coefficient comparison above was possible as $\cos nx$ & $\sin nx$ & 1 are orthogonal on a 2π range $\left(\int_0^{2\pi} f(x)g(x)\,dx = 0 \text{ if } f(x) \neq g(x)\right)$

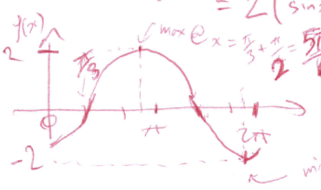
(d) See (b)



(e) From (b) $f(x) = \sin x - \sqrt{3}\cos x = 2\left(\sin x \cdot \frac{1}{2} - \cos x\frac{\sqrt{3}}{2}\right)$

$= 2\left(\sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3}\right) = 2\sin\left(x - \frac{\pi}{3}\right)$

(f)



max @ $x = \frac{\pi}{3} + \frac{\pi}{2} = \frac{5\pi}{6}$

min @ $x = \frac{5\pi}{6} + \pi = \frac{11\pi}{6}$

(h) Using periodicity

$$\int_0^{2\pi}\left(2\sin\left(x - \frac{\pi}{3}\right)\right)^2 dx = \int_0^{2\pi}(2\sin x)^2 dx$$
$$= 4\int_0^{2\pi}\sin^2 dx = 4\pi.$$

□.

15S

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Solution(s):

From user: ar857

(15) a) $a \wedge (b \wedge c) = (a \cdot c)b - (a \cdot b)c$

b) $(a \wedge b) \wedge (c \wedge d) = c((a \wedge b) \cdot d) - ((a \wedge b) \cdot c)d$

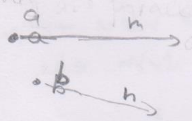
$= ((c \wedge d) \cdot a)b - ((c \wedge d) \cdot b)a$

$-a[b,c,d] + b[a,c,d] = c[a,b,d] - d[a,b,c]$

$a[b,c,d] - b[a,c,d] + c[a,b,d] - d[a,b,c] = 0$

c) $r = a + sm$ $r = b + tn$ these lines intersect if they are coplanar.

all 3 vectors m, n and $(a-b)$ must lie in the same plane therefore



$(a-b) \cdot (m \times n) = 0 = |a-b||m \times n|\cos\theta = 0 = [(a-b), m, n]$

$a-b$ must be $\perp$ to $m \times n$ so $\cos\theta = \cos 90° = 0$

d)

$a + sm = b + tn \; / \cdot (n \times a)$ $\quad a \cdot m \times n = n \cdot a \times m = m \cdot n \times a$

$a \cdot (n \times a) + s m \cdot (n \times a) = b \cdot (n \times a) + t n(n \times a)$

$s = \frac{[b,n,a]}{[m,n,a]} = \frac{[b,n,a]}{[a,m,n]}$

$a + sm = b + tn \; / \cdot (a \times m)$

$0 + 0 = b \cdot a \times m + t n \cdot a \times m$

$t = \frac{[b\ m\ a]}{[n\ a\ m]} = \frac{[b,m,a]}{[a,m,n]}$

$h = \frac{dy}{dx} \quad \frac{dh}{dx} = \frac{d^2y}{dx^2}$

16T

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Solution(s):

From user: ar857

2011 16 T

a) $xy'' + y' = 0$ $\quad u = \frac{dy}{dx}$ $\Rightarrow \frac{d}{dx}(u) = \frac{du}{dx}$

$x\frac{du}{dx} + u = 0$

$-\frac{1}{u}du = \frac{1}{x}dx$

$\frac{1}{u} = Kx$

$B\frac{1}{x} = \frac{dy}{dx} \Rightarrow y = B\ln x + C$ $\quad B = 2$, $C = -1$

$y = 2\ln x - 1$

c) $y'' + 4y = \sin x + \cos x$

$y_c = D_1 \sin 2x + D_2 \cos 2x$

$y_p = a\sin x + b\cos x \Rightarrow a, b = 1/3$

$y(0) = D_2 + 1/3 = 3 \Rightarrow D_2 = 8/3$

$y'(0) = D_1 + \frac{\sqrt{2}}{3} = 0 \Rightarrow D_1 = -\frac{\sqrt{2}}{3}$

$y = 8/3 \sin 2x - \frac{\sqrt{2}}{3}\cos 2x + 1/3(\sin x + \cos x)$

b) $y' + 2xy = 2xe^{-x^2}$ $\quad e^{\int 2x} = e^{x^2}$

$e^{x^2}y = x^2 + c \Rightarrow y = (x^2 - 1)e^{-x^2}$

From user: ip343

(c) $\frac{d^2y}{dx^2} + 4y = \sin x + \cos x$

$\underbrace{\left(\frac{d^2}{dx^2} + 4\right)}_{L} y = \sin x + \cos x$

Auxiliary eq'n: $\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

So, complementary eq'n: $y_c = \tilde{A} e^{2ix} + \tilde{B} e^{-2ix} = A \cos 2x + B \sin 2x$

Try particular integral $y_{PI} = p \sin x + q \cos x$

$\frac{dy_{PI}}{dx} = p \cos x - q \sin x$

$\frac{d^2 y_{PI}}{dx^2} = -p \sin x - q \cos x$

So, $L y_{PI} = -p \sin x - q \cos x + 4(p \sin x + q \cos x) = \sin x + \cos x$.

Comparing coefficients of $\sin x$: | $\cos x$:

$-p + 4p = 1$ | $-q + 4q = 1$

$p = \frac{1}{3}$ | $q = \frac{1}{3}$

So, $y = \cancel{\frac{1}{3}} A \cos 2x + B \sin 2x + \frac{1}{3}(\sin x + \cos x)$

Impose boundary conditions:

(i) $y|_{x=0} = 3 = A + \frac{1}{3} \quad \Rightarrow \quad A = \frac{8}{3}$

$y|_{x=\frac{\pi}{4}} = 0 = B + \frac{1}{3}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) \quad \Rightarrow \quad B = -\frac{\sqrt{2}}{3}$

$\therefore y = \frac{8}{3} \cos 2x - \frac{\sqrt{2}}{3} \sin 2x + \frac{1}{3}(\sin x + \cos x)$

17Z

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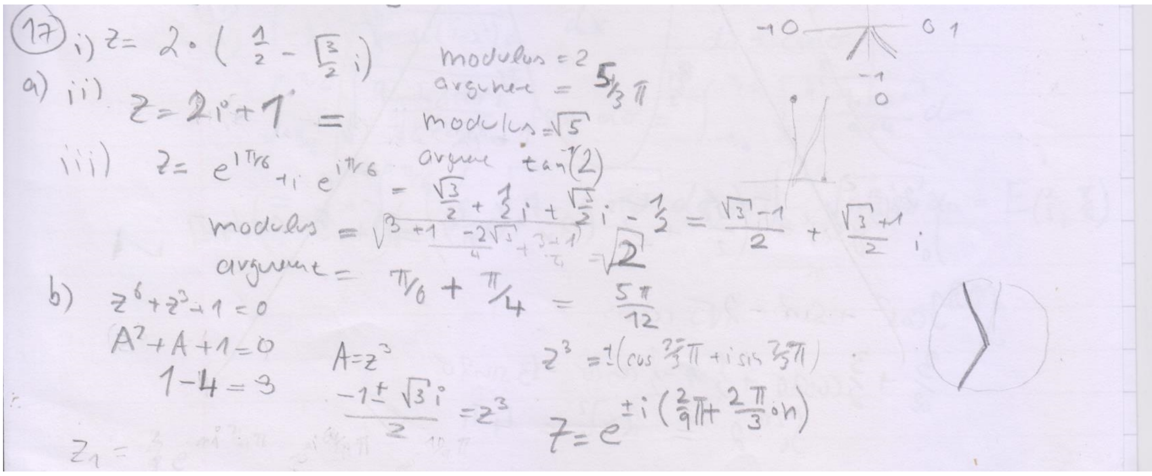
Solution(s):

From user: ar857



(17) i) $z = 2 \cdot \left(\frac{1}{2} - \frac{\sqrt{3}}{2} i \right)$ modulus $= 2$ argument $= 5/3\,\pi$

a) ii) $z = 2i + 1 =$ modulus $= \sqrt{5}$ argument $\tan^{-1}(2)$

iii) $z = e^{i\pi/6} + i e^{i\pi/6} = \frac{\sqrt{3}}{2} + \frac{1}{2} i + \frac{\sqrt{3}}{2} i - \frac{1}{2} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2} i$

modulus $= \sqrt{\frac{3+1-2\sqrt{3}}{4} + \frac{3+1}{4}} = \sqrt{2}$

argument $= \pi/6 + \pi/4 = \frac{5\pi}{12}$

b) $z^6 + z^3 + 1 = 0$

$A^2 + A + 1 = 0$ $\quad A = z^3$ $\quad z^3 = 1\left(\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi\right)$

$1 - 4 = 3$ $\quad \frac{-1 \pm \sqrt{3} i}{2} = z^3$ $\quad z = e^{\pm i \left(\frac{2}{9}\pi + \frac{2\pi}{3} \cdot n\right)}$

$z_1 =$

18Z

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Solution(s):

From user: ar857

2011 I B 18

a) to be exact $\frac{\partial}{\partial y}(p(x,y)) = \frac{\partial}{\partial x}(q(x,y))$

b) i) $2\sin x \cosh y\, dx + \sin^2 x \sinh y\, dy$ not exact

ii) $\sinh(x+iy)\, dx - \sin(y-ix)\, dy$

$A = i\cosh(x+iy) = i\cosh x \cos iy + i\sinh x \sinh iy$

$B = i\cos(y-ix) = i\cos y \cos ix + i\sin y \sin ix$

$= i\cos iy \quad \cosh x \quad \sinh iy\, i\sinh x$

$A = B \Rightarrow$ ~~ex~~ ii) is exact

c) $(\cos x + y\sin x)\,dx + x\sin x\,dy = 0$

$$\frac{\sin x - x\cos x - \sin x}{x\sin x} = \frac{1}{\mu}\frac{d\mu}{dx}$$

$$-\cot g\, x \; dx = \frac{1}{\mu}\, d\mu$$

$$\ln\mu = -\ln|\sin x|$$

$$\mu = \frac{1}{\sin x}$$

$$\left(\frac{\cos x}{\sin x} + y\right)dx \quad x\,dy = 0$$

$$\ln|\sin x| + yx = D$$

$$y = \frac{D}{x} - \frac{\ln|\sin x|}{x}$$

$$\left.\frac{\partial a}{\partial b}\right|_c \left.\frac{\partial b}{\partial d}\right|_e + \left.\frac{\partial c}{\partial d}\right|_e \left.\frac{\partial a}{\partial c}\right|_b =$$

d)

$$\left(\frac{\partial a}{\partial b}\right)_c\left(\frac{\partial b}{\partial d}\right)_e + \left(\frac{\partial a}{\partial c}\right)_b\left(\frac{\partial c}{\partial d}\right)_e = \left(\frac{\partial a}{\partial d}\right)_e$$

$$a(b,c) = a(b(d,e), c(d,e)) = a(d,e)$$

$$db = \left(\frac{\partial b}{\partial d}\right)_e dd + \left(\frac{\partial b}{\partial e}\right)_d de$$

$$dc = \left(\frac{\partial c}{\partial d}\right)_e dd + \left(\frac{\partial c}{\partial e}\right)_d de$$

$$da = \left(\frac{\partial a}{\partial b}\right)_c db + \left(\frac{\partial a}{\partial c}\right)_b dc$$

$$da = \left(\left(\frac{\partial a}{\partial b}\right)_c \left(\frac{\partial b}{\partial d}\right)_e + \left(\frac{\partial a}{\partial c}\right)_b \left(\frac{\partial c}{\partial d}\right)_e\right) dd \Big\} = \left(\frac{\partial a}{\partial d}\right)_e dd$$

$$+ \left(\left(\frac{\partial a}{\partial b}\right)_c \left(\frac{\partial c}{\partial e}\right)_d + \left(\frac{\partial a}{\partial c}\right)_b \left(\frac{\partial c}{\partial e}\right)_d\right) de \Big\} = \left(\frac{\partial a}{\partial e}\right)_d de$$

19R*

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Solution(s):

From user: ip343

ES 3: Scalar and vector fields — 2011 Paper 1 19R*

Tripos Questions

T1. (a) $\oint_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = \int_V (\underset{\sim}{\nabla} \cdot \underset{\sim}{F})\, dV$ — Divergence theorem.

$= \int_V \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (y^2, e^{xz}, e^{xy})\, dV$

$= \int_V (0+0+0)\, dV = 0$ ✓

(b) $\int_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = \int_S \underset{\sim}{F} \cdot \underset{\sim}{n}\, dS$

$= \int_S \underset{\sim}{F} \cdot (0,1,0)\, dS$

$= \int_S (y+1) e^{-(x^2+\sin^2 y+z^2)} dS \,\Big|_{y=0}$ ✓ FINE.

(*) $= \int_S e^{-x^2-z^2} dS$

$= \int_{x=-\infty}^{\infty} \int_{z=-\infty}^{\infty} e^{-x^2-z^2}\, dx\, dz$

$= \int_{x=-\infty}^{\infty} \int_{z=-\infty}^{\infty} e^{-x^2} \cdot e^{-z^2}\, dx\, dz$

$= \int_{x=-\infty}^{\infty} \sqrt{\pi}\, e^{-x^2} dx$

$= \sqrt{\pi} \times \sqrt{\pi} = \pi$ ✓

OR, IN POLARS: $x = r\cos\theta$, $z = r\sin\theta$

COULD DO

(*) $= \iint e^{-r^2} r\,dr\,d\theta$ (over plane)

$= 2\pi \left[-\frac{1}{2} e^{-r^2}\right]_0^{\infty}$

from $\int d\theta$

$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

$= 2\pi \left(0 - \left(-\frac{1}{2}\right)\right)$

$= \pi$

(c) Equation of sphere: $(x-\pi)^2 + (y-\sqrt{2})^2 + (z-e)^2 = r^2$

$\int_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = \int_V (\underset{\sim}{\nabla} \cdot \underset{\sim}{F})\, dV$ — Divergence theorem

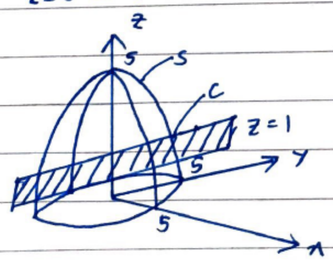
$= \int_V \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (kx, ky, kz)\, dV$

$= 3k \int_V dV$ — volume of a sphere

$= 3k\left(\frac{4}{3}\pi r^3\right)$

$= 4k\pi r^3$ ✓ super.

(d) $z = 5 - x^2 - y^2$



~~$\int_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = \int_S (\underset{\sim}{\nabla} \times \underset{\sim}{A}) \cdot d\underset{\sim}{S}$~~

~~$= \int_S \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (yz^2, -3xy, x^3y^3) \cdot d\underset{\sim}{S}$~~

~~$= \int_S (3x^3y^2, 2yz - 3x^2y^3, -3y - 3z) \cdot d\underset{\sim}{S}$~~

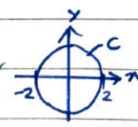
$\int_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = \int_S (\underset{\sim}{\nabla} \times \underset{\sim}{A}) \cdot d\underset{\sim}{S}$ — Stokes' theorem

$= \int_C \underset{\sim}{A} \cdot d\underset{\sim}{x}$

$= \int_C \underset{\sim}{A} \cdot \frac{dx}{d\theta} \cdot d\theta$

At $z = 1$, $1 = 5 - x^2 - y^2$

$x^2 + y^2 = 4$ ✓



✓

$= \int_C (yz^2, -3xy, x^3y^3) \cdot (-2\sin\theta, 2\cos\theta, 0)\, d\theta$

$= \int_{\theta=0}^{2\pi} (2\sin\theta \cdot 1, -8\cos\theta\sin\theta, 2^6\sin^3\theta\cos^3\theta) \cdot (-\sin\theta, \cos\theta, 0)\, d\theta$

$\underset{\sim}{x} = (2\cos\theta, 2\sin\theta, 0)$

$\frac{d\underset{\sim}{x}}{d\theta} = (-2\sin\theta, 2\cos\theta, 0)$

From user: ip343

$$\int_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = \int_S (\underset{\sim}{\nabla} \times \underset{\sim}{A}) \cdot d\underset{\sim}{S}$$

$\underset{\sim}{x} = (2\cos\theta, 2\sin\theta, 1)$

$$= \int_C \underset{\sim}{A} \cdot d\underset{\sim}{x}$$

$\frac{d\underset{\sim}{x}}{d\theta} = (-2\sin\theta, 2\cos\theta, 0)$

$$= \int_C \underset{\sim}{A} \cdot \frac{d\underset{\sim}{x}}{d\theta} d\theta$$

$$= \int_C (yz^2, -3xy, x^3y^3) \cdot (-2\sin\theta, 2\cos\theta, 0)\, d\theta$$

$$= \int_{\theta=0}^{2\pi} (-4\sin^2\theta - 24\sin\theta\cos^2\theta)\, d\theta$$

$$= \int_{\theta=0}^{2\pi} -4\sin^2\theta\, d\theta + \int_{\theta=0}^{2\pi} -24\sin\theta\cos^2\theta\, d\theta$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$= \int_{\theta=0}^{2\pi} (2\cos 2\theta - 2)\, d\theta - 24\left[-\tfrac{1}{3}\cos^3\theta\right]_0^{2\pi}$$

$$= \left[\sin 2\theta - 2\theta\right]_0^{2\pi} + \underbrace{8(\cos^3 2\pi - \cos^3 0)}_{0}$$

$$= (\sin 4\pi - 4\pi) - 0$$

$$= -4\pi$$

20X*

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