2010 Mathematics (1)

Section A

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Section B

11S

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12X

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13Y

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14Z

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15S

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16T

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17T

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18X

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Solution(s):

From user: mz407

2010A. 18X

$$u(x,t) = t^a y(b^a t^a x)$$

a) $\frac{\partial u}{\partial t}\Big|_x = a t^{a-1} y + t^a \frac{\partial y}{\partial t}\Big|_x$

$\frac{\partial^3 u}{\partial x^3}\Big|_t = t^a \frac{\partial^3 y}{\partial x^3}\Big|_t$

b). $\boxed{\frac{d^2 y}{ds^2} - sy = 0 \implies \frac{\partial u}{\partial t}\Big|_x + \frac{\partial^3 u}{\partial x^3}\Big|_t = 0.}$

$\frac{\partial u}{\partial t}\Big|_x + \frac{\partial^3 u}{\partial x^3}\Big|_t = a t^{a-1} y + t^a \frac{\partial y}{\partial t}\Big|_x + t^a \frac{\partial^3 y}{\partial x^3}\Big|_t$ (*)

~~$= t^a\left(a t^{-1} y + \frac{\partial y}{\partial t}\Big|_x + \frac{\partial^3 y}{\partial x^3}\Big|_t\right)$~~ (*)

$\frac{\partial y}{\partial t}\Big|_x = \frac{dy}{ds} \cdot \frac{\partial s}{\partial t}\Big|_x$ $\qquad \frac{\partial s}{\partial x}\Big|_t = b^a t^a \qquad \frac{\partial^2 s}{\partial x^2}\Big|_t = 0$

$\frac{\partial s}{\partial t}\Big|_x = b^a x a t^{a-1}$

$\frac{\partial^3 y}{\partial x^3}\Big|_t = \frac{\partial}{\partial x}\Big|_t \frac{\partial}{\partial x}\Big|_t \frac{\partial y}{\partial x}\Big|_t$

$= \frac{\partial}{\partial x}\Big|_t \frac{\partial}{\partial x}\Big|_t \left(\frac{dy}{ds} \cdot \frac{\partial s}{\partial x}\Big|_t\right)$

$= \frac{\partial}{\partial x}\Big|_t \left(\frac{\partial s}{\partial x}\Big|_t \left(\frac{d^2 y}{ds^2} \cdot \frac{\partial s}{\partial x}\Big|_t\right) + \frac{dy}{ds} \cdot \frac{\partial^2 s}{\partial x^2}\Big|_t \right)$

$= \frac{\partial}{\partial x}\Big|_t \frac{d^2 y}{ds^2} \cdot \left(\frac{\partial s}{\partial x}\Big|_t\right)^2$

$= \frac{d^3 y}{ds^3} \cdot \frac{\partial s}{\partial x}\Big|_t \left(\frac{\partial s}{\partial x}\Big|_t\right)^2 + 2 \frac{\partial s}{\partial x}\Big|_t \cdot \frac{\partial^2 s}{\partial x^2}\Big|_t \cdot \frac{d^2 y}{ds^2}$

$= \frac{d^3 y}{ds^3} \cdot \left(\frac{\partial s}{\partial x}\Big|_t\right)^3$

$\frac{d^2 y}{ds^2} - sy = 0$

$\Rightarrow \frac{d^3 y}{ds^3} - y - s\frac{dy}{dx} = 0 \Rightarrow \frac{d^3 y}{ds^3} = y + s\frac{dy}{ds}$

(*) ~~$= t^a\left(a t^{-1} y + b^a x a t^{a-1} \frac{dy}{ds} + \left(y + s\frac{dy}{ds}\right) b^{3a} t^{3a}\right)$~~

$= a t^{a-1} y + t^a b^a x a t^{a-1} \frac{dy}{ds} + t^a \left(y + s\frac{dy}{ds}\right) t^{3a} b^{3a}$

$= a t^{a-1}\left(y + t^a b^a x \frac{dy}{ds}\right) + t^{4a} b^{3a}\left(y + t^a b^a x \frac{dy}{ds}\right)$

$= \left(a t^{a-1} + t^{4a} b^{3a}\right)\left(y + s\frac{dy}{ds}\right)$

$= 0$ if $a t^{a-1} + b^{3a} t^{4a} = 0 \Rightarrow \begin{cases} a = -b^{3a} \\ a - 1 = 4a \end{cases} \Rightarrow \begin{cases} a = -\frac{1}{3} \\ b = +3 \end{cases}$

$$u = t^a \, y\left(b^a t^a x\right)$$

(a) (i) $$\frac{\partial u}{\partial t} = a t^{a-1} y + a b^a t^{a-1} x y'$$

(ii) $$\frac{\partial^3 u}{\partial x^2} = t^a \left(b^a t^a\right)^3 y''' = t^{4a} b^{3a} y'''$$

(b) Now we are told to assume that $\boxed{y'' = sy}$.

This fact $\Rightarrow$ $y''' = sy' + y$

$$\therefore \frac{\partial^3 u}{\partial x^3} + \frac{\partial u}{\partial t} = t^{4a} b^{3a} \big(\underbrace{b^a t^a x}_{\text{"}s\text{"}} \, y' + y\big) + a t^{a-1} y + a b^a t^{a-1} x y'$$

$$= \left(t^{5a} b^{4a} + a t^{a-1} b^a\right) x y' + \left(t^{4a} b^{3a} + a t^{a-1}\right) y$$

$$= \left(t^{4a} b^{3a} + a t^{a-1}\right)\left(b^a x y' + y\right)$$

For RHS to be equal to zero and independent of t we could require $\left(t^{4a} b^{3a} + a t^{a-1} = 0 \quad \forall\, t\right)$

$$\Leftarrow \begin{cases} 4a = a-1 & \text{(same } t \text{ powers)} \\ b^{3a} = -a & \text{(cancellation)} \end{cases} \Leftarrow \begin{cases} a = -1/3 \\ b = 3 \end{cases}. \quad \text{QED.}$$

19Z*

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20Y*

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