

# 2010 Mathematics (1)

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## Section A

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## **Section B**

**11S**

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**12X**

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**13Y**

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**14Z**

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**15S**

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**16T**

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**17T**

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**18X**

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***Solution(s):***

From user: mz407

2010A. 18x

$$u(x,t) = t^a y(b^a t^a x)$$

$$a) \quad \frac{\partial u}{\partial t} \Big|_x = a t^{a-1} y + t^a \frac{\partial y}{\partial t} \Big|_x$$

$$\frac{\partial^2 u}{\partial x^2} \Big|_t = t^a \frac{\partial^2 y}{\partial x^2} \Big|_t$$

$$b) \quad \left[ \frac{d^2 y}{ds^2} - sy = 0 \implies \frac{\partial u}{\partial t} \Big|_x + \frac{\partial^2 u}{\partial x^2} \Big|_t = 0 \right]$$

$$\frac{\partial u}{\partial t} \Big|_x + \frac{\partial^2 u}{\partial x^2} \Big|_t = a t^{a-1} y + t^a \frac{\partial y}{\partial t} \Big|_x + t^a \frac{\partial^2 y}{\partial x^2} \Big|_t$$

~~$$t^a (a t^{-1} y + \frac{\partial y}{\partial t} \Big|_x + \frac{\partial^2 y}{\partial x^2} \Big|_t) = 0$$~~

$$\frac{\partial y}{\partial t} \Big|_x = \frac{dy}{ds} \cdot \frac{\partial s}{\partial t} \Big|_x$$

$$\frac{\partial s}{\partial t} \Big|_x = b^a t^a$$

$$\frac{\partial s}{\partial x} \Big|_t = 0$$

$$\frac{\partial s}{\partial t} \Big|_x = b^a x a t^{a-1}$$

$$\frac{\partial^2 y}{\partial x^2} \Big|_t = \frac{\partial}{\partial x} \Big|_t \frac{\partial}{\partial x} \Big|_t \frac{dy}{ds} \Big|_t$$

$$= \frac{\partial}{\partial x} \Big|_t \frac{\partial}{\partial x} \Big|_t \left( \frac{dy}{ds} \cdot \frac{\partial s}{\partial x} \Big|_t \right)$$

$$= \frac{\partial}{\partial x} \Big|_t \left[ \frac{\partial s}{\partial x} \Big|_t \left( \frac{d^2 y}{ds^2} \cdot \frac{\partial s}{\partial x} \Big|_t \right) + \frac{dy}{ds} \cdot \frac{\partial^2 s}{\partial x^2} \Big|_t \right]$$

$$= \frac{\partial}{\partial x} \Big|_t \frac{d^2 y}{ds^2} \cdot \left( \frac{\partial s}{\partial x} \Big|_t \right)^2$$

$$= \frac{d^2 y}{ds^2} \cdot \frac{\partial s}{\partial x} \Big|_t \left( \frac{\partial s}{\partial x} \Big|_t \right)^2 + 2 \frac{\partial s}{\partial x} \Big|_t \cdot \frac{\partial^2 s}{\partial x^2} \Big|_t \cdot \frac{dy}{ds}$$

$$= \frac{d^2 y}{ds^2} \cdot \left( \frac{\partial s}{\partial x} \Big|_t \right)^3$$

$$\frac{d^2 y}{ds^2} - sy = 0$$

$$\Rightarrow \frac{d^2 y}{ds^2} - y - s \frac{dy}{ds} = 0 \Rightarrow \frac{d^2 y}{ds^2} = y + s \frac{dy}{ds}$$

~~$$= t^a (a t^{-1} y + b^a x a t^{a-1} \frac{dy}{ds} + (y + s \frac{dy}{ds}) b^{2a} t^{2a})$$~~

$$= a t^{a-1} y + t^a b^a x a t^{a-1} \frac{dy}{ds} + t^a (y + s \frac{dy}{ds}) t^{2a} b^{2a}$$

~~$$= a t^{a-1} y + t^a b^a x a t^{a-1} \frac{dy}{ds} + t^a b^{2a} (y + s \frac{dy}{ds}) t^{2a}$$~~

$$= (a t^{a-1} + t^{4a} b^{2a}) (y + s \frac{dy}{ds})$$

$$= 0 \quad \text{if} \quad a t^{a-1} + b^{2a} t^{4a} = 0 \Rightarrow \begin{cases} a = -b^{2a} \\ a-1 = 4a \end{cases} \Rightarrow \begin{cases} a = -\frac{1}{3} \\ b = +3 \end{cases}$$

$$u = t^a y(b^a t^a x)$$

$$(a) \quad (i) \quad \frac{\partial u}{\partial t} = a t^{a-1} y + a b^a t^{a-1} x y'$$

$$(ii) \quad \frac{\partial^3 u}{\partial x^3} = t^a (b^a t^a)^3 y''' = t^{4a} b^{3a} y'''$$

(b) Now we are told to assume that  $\boxed{y'' = sy'}$ .

$$\text{This fact} \Rightarrow y''' = s y' + y$$

$$\begin{aligned} \therefore \frac{\partial^3 u}{\partial x^3} + \frac{\partial u}{\partial t} &= t^{4a} b^{3a} (\underbrace{b^a t^a}_{"s"} y' + y) + a t^{a-1} y + a b^a t^{a-1} x y' \\ &= (t^{5a} b^{4a} + a t^{a-1} b^a) x y' + (t^{4a} b^{3a} + a t^{a-1}) y \\ &= (t^{4a} b^{3a} + a t^{a-1}) (b^a x y' + y) \end{aligned}$$

For RHS to be equal to zero and independent of  $t$  we could require  $(t^{4a} b^{3a} + a t^{a-1} = 0 \quad \forall t)$

$$\Leftrightarrow \left\{ \begin{array}{ll} 4a = a-1 & (\text{same } t \text{ powers}) \\ b^{3a} = -a & (\text{cancellation}) \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} a = -\frac{1}{3} \\ b = 3 \end{array} \right\}. \text{ QED.}$$

**19Z\***

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**20Y\***

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