

## 2009 Mathematics (2)

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### Section A

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## **Section B**

**11S**

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**12T**

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***Solution(s):***

From user: ar857

2009 II 12

i)  $y'' + \lambda y' + ky = 0$

$$A^2 + \lambda A + k = 0 \quad A_{1,2} = \frac{-\lambda \pm \sqrt{\lambda^2 - 4k}}{2}$$

$$y = c_1 e^{A_1 t} + c_2 e^{A_2 t}$$

Oscillating if  $4k > \lambda^2$

ii)  $y = e^{-\frac{\lambda}{2}t} \cos \sqrt{(4k - \lambda^2)/4} t + c_2 \sin \sqrt{(4k - \lambda^2)/4} t \Rightarrow \omega_0 = \sqrt{\frac{4k - \lambda^2}{4}}$

$$e^{-\frac{\lambda}{2}t} = 0.75$$

$$\frac{\lambda}{2} = e^{\frac{1}{2}t} \Rightarrow \frac{1}{3} = e^{\frac{1}{2}t} \Rightarrow \frac{1}{3} = \frac{2}{\lambda} \ln \frac{1}{3}$$

iii)  $y'' + w^2 y = 2te^{-t}$

$$\lambda^2 + w^2 = 0 \quad \lambda = \pm wi$$

$$y_c = D_1 \cos wt + D_2 \sin wt \quad D_1 = 0$$

$$y_p = (Kt + B)e^{-t}$$

$$L[y_p] = K \frac{1}{s^2} + B \frac{1}{s} = \frac{K}{s^2} + \frac{B}{s}$$

$$= \frac{K}{s^2} + \frac{B}{s} = \frac{K + Bs^2}{s^2}$$

$$= \frac{K}{s^2} + \frac{Bs^2}{s^2} = \frac{K + Bs^2}{s^2}$$

$$= \frac{K}{s^2} + \frac{Bs^2}{s^2} = \frac{K + Bs^2}{s^2}$$

$$\Rightarrow K = \frac{2}{1+w^2} \quad B = \frac{4}{(1+w^2)^2}$$

$$y = D_1 \cos wt + D_2 \sin wt + \frac{4}{(1+w^2)^2} e^{-t} + \frac{2}{(1+w^2)^2} t e^{-t} + \frac{2}{(1+w^2)^2} t^2 e^{-t}$$

$$y(0) = D_1 + 0 + \frac{4}{(1+w^2)^2} = 0 \Rightarrow D_1 = -\frac{4}{(1+w^2)^2}$$

$$y'(0) = w D_1 \cos 0 + w D_2 \sin 0 + \frac{4}{(1+w^2)^2} + \frac{2}{(1+w^2)^2} = 0$$

$$= 0 + w D_2 - \frac{2}{(1+w^2)^2} + 2w \Rightarrow D_2 = \frac{2(1-w^2)}{(1+w^2)^2}$$

$$= 0 + w D_2 - \frac{2}{(1+w^2)^2} + 2w \Rightarrow D_2 = \frac{2(1-w^2)}{(1+w^2)^2}$$

$$D_2 = \frac{2(1-w^2)}{(1+w^2)^2}$$

$$y = \frac{1}{(1+w^2)^2} \left( -4 \cos wt + \frac{2-2w^2}{w} \sin wt + 4e^{-t} + 2te^{-t} + 2w^2 t^2 e^{-t} \right)$$

$$\text{for } t \rightarrow \infty \quad A = \sqrt{16 + \frac{(2-2w)^2}{w^2}}$$

13X

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14Y

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**15R**

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**16S**

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**17X**

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**18Y**

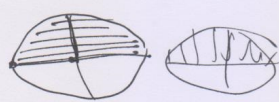
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**Solution(s):**

From user: ar857

2009 184

a)  $\int_0^1 \int_0^2 x e^{xy} dx dy = \int_0^2 e^{xy} \Big|_0^1 dy = \int_0^2 (e^y - 1) dy = e^2 - 2 - 1 = e^2 - 3$

b)  $\int_0^{\frac{1}{2}} \int_{-\sqrt{1-4y^2}}^{\sqrt{1-4y^2}} y dx dy$   $x^2 = 1 - 4y^2$   $y^2 = \frac{1-x^2}{4}$  

$= \int_{-1}^1 \int_0^{\sqrt{1-x^2}} y dy dx = \int_{-1}^1 \frac{1}{2} (1-x^2) dx = \frac{1}{2} \cdot \left(1 - \frac{1}{3} - \frac{1}{3} + 1\right) = \frac{1}{2} \cdot \frac{4}{3} = \frac{1}{3}$

c)  $0 \leq r \leq 1$   $0 \leq \theta \leq \frac{1}{2}\pi$   $x+y+z=4$   $z=0 \Rightarrow 4-x-y \leq z \leq 0$

$\iiint_V dx dy dz = \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{4-r(\cos\theta+\sin\theta)} r dz dr d\theta$

$= \int_0^{\frac{\pi}{2}} \int_0^1 4r - r^2(\cos\theta + \sin\theta) dr d\theta = \int_0^{\frac{\pi}{2}} 2 - \frac{\cos\theta + \sin\theta}{3} d\theta = 2\left[\frac{\pi}{2}\right] - \frac{1}{3}(1+0-0+1) = \pi - \frac{2}{3}$

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**19R\***

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**20T\***

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