2009 Mathematics (2)

Section A

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Section B

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12T

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Solution(s):

From user: ar857

2009 II 12

i) $y'' + \lambda y' + ky = 0$

$A^2 + \lambda A + k = 0 \qquad A_{1,2} = \frac{\pm\sqrt{\lambda^2 - 4k} - \lambda}{2}$

$y = c_1 e^{A_1 t} + c_2 e^{A_2 t}$

oscillatory if $4k > \lambda^2$

ii) $y = e^{-\frac{\lambda}{2}t}\left(c_1 \cos\sqrt{(4k-\lambda^2)/2} + c_2 \sin\sqrt{(4k-\lambda^2)/2}\right) \Rightarrow w_0 = \sqrt{\frac{4k-\lambda^2}{2}}$

$e^{-\frac{\lambda}{2}t} = 0.75$

$\frac{4}{3} = e^{\frac{\lambda}{2}t} \Rightarrow t = \frac{2}{\lambda}\ln\frac{4}{3}$

iii) $y'' + w^2 y = 2te^{-t}$

$\lambda^2 + w^2 = 0 \qquad \lambda = \pm wi$

$y_c = D_1 \cos wt + D_2 \sin wt \qquad D_1 = 0$

$y_p = (Kt + B)e^{-t}$

$\Rightarrow K = \frac{2}{1+w^2} \qquad B = \frac{4}{(1+w^2)^2}$

$y = D_1 \cos wt + D_2 \sin wt + \frac{4}{(1+w^2)^2}e^{-t} + \frac{2te^{-t}}{(1+w^2)^2} + \frac{2w^2te^{-t}}{(1+w^2)^2}$

$\frac{-2(\omega-1)}{c(\omega^2-1)^2}$

$y(0) = D_1 + 0 + \frac{4}{(1+w^2)^2} = 0 \rightarrow D_1 = -\frac{4}{(1+w^2)^2}$

$y'(0) = -wD_1 \sin 0 + wD_2 \cos 0 - \frac{4}{(1+w^2)^2} + 2 + 2w^2 = 0$

$= 0 + wD_2 - \frac{2}{(1+w^2)^2} + 2w^2 \Rightarrow D_2 = \left(\frac{2}{(1+w^2)^2} - 2w^2\right)/w$

$D_2 = \frac{2(1-w^2)}{(1+w^2)^2 \cdot w}$

$y = \frac{1}{(1+w^2)^2}\cdot\left(-4\cos wt + \frac{2-2w^2}{w}\sin wt + 4e^{-t} + 2te^{-t} + 2w^2te^{-t}\right)$

for $t \to \infty$ $\quad A = \sqrt{16 + \frac{(2-2w)^2}{w^2}}$

13X

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14Y

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15R

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16S

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17X

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18Y

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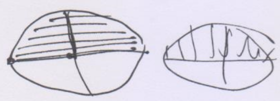
Solution(s):

From user: ar857

2009 18 4

a) $\int_0^1\int_0^2 xe^{xy}\,dx\,dy = \int_0^2 e^{xy}\Big]_0^1 dx = \int_0^2 (e^x-1)\,dx = e^2-2-1 = e^2-3$

b) $\int_0^{1/2}\int_{-\sqrt{1-4y^2}}^{\sqrt{1-4y^2}} y\,dx\,dy$ $\quad x^2 = 1-4y^2$, $\quad y^2 = \frac{1-x^2}{4}$



$= \int_{-1}^{1}\int_0^{\sqrt{\frac{1-x^2}{4}}} y\,dy\,dx = \int_{-1}^{1}\frac{1}{8}(1-x^2)\,dx = \frac{1}{8}\cdot\left(1-\frac{1}{3}-\frac{1}{3}+1\right) = \frac{4}{3}\cdot\frac{1}{8} = \frac{1}{6}$

c) $0 \le r \le 1 \quad 0 \le \phi \le \frac{1}{2}\pi \quad x+y+z=4,\ z=0 \Rightarrow 4-x-y \le z \le 0$

$\iiint_V dx\,dy\,dz = \int_0^{\pi/2}\int_0^1\int_0^{4-r(\cos\phi+\sin\phi)} r\,dz\,dr\,d\phi$

$= \int_0^{\pi/2}\int_0^1 4r - r^2(\cos\phi+\sin\phi)\,dr\,d\phi = \int_0^{\pi/2} 2-\frac{\cos\phi+\sin\phi}{3} = 2\frac{\pi}{2}-\frac{1}{3}(1+0-0+1) = \pi-\frac{2}{3}$

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19R*

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