

2009 Mathematics (2)

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Section A

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Section B

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Solution(s):

From user: ar857

2009 II 12

i) $y'' + \lambda y' + ky = 0$

$A^2 + \lambda A + k = 0 \quad A_{1,2} = \frac{-\lambda \pm \sqrt{\lambda^2 - 4k}}{2}$

$y = c_1 e^{A_1 t} + c_2 e^{A_2 t}$

oscillatory if $4k > \lambda^2$

ii) $y = c_1 \cos \sqrt{(4k - \lambda^2)/2} t + c_2 \sin \sqrt{(4k - \lambda^2)/2} t \Rightarrow \omega_0 = \sqrt{\frac{4k - \lambda^2}{2}}$

$e^{-\lambda/2 t} = 0.75$

$\frac{4}{3} = e^{\lambda/2 t} \Rightarrow \frac{4}{3} = \frac{2}{\lambda} \ln \frac{4}{3}$

iii) $y'' + w^2 y = 2te^{-t}$

$\lambda^2 + w^2 = 0 \quad \lambda = \pm wi$

$y_c = D_1 \cos wt + D_2 \sin wt \quad D_1 = 0$

~~$y_p = Kte^{-t} + Bte^{-t}$~~ $y_p = (Kt+B)e^{-t}$

~~$L[y_p] = Kte^{-t} + Bte^{-t} + w^2(Kte^{-t} + Bte^{-t}) = 2te^{-t}$~~

~~$-2Kte^{-t} + w^2 Kte^{-t} + w^2 Bte^{-t} = 2te^{-t}$~~

~~$-Kte^{-t} - Kte^{-t} + w^2 Kte^{-t} + w^2 Bte^{-t} = 2te^{-t}$~~

~~$Ky_p = Kte^{-t} - Kte^{-t} + w^2 Kte^{-t} + w^2 Bte^{-t} = 2te^{-t}$~~

$\Rightarrow K = \frac{2}{1+w^2} \quad B = \frac{4}{(1+w^2)^2}$

$y = D_1 \cos wt + D_2 \sin wt + \frac{4}{(1+w^2)^2} e^{-t} + \frac{2}{(1+w^2)^2} te^{-t} + \frac{2w^2 te^{-t}}{(1+w^2)^2}$

$y(0) = D_1 + 0 + \frac{4}{(1+w^2)^2} = 0 \Rightarrow D_1 = -\frac{4}{(1+w^2)^2}$

$y'(0) = wD_2 + wD_1 \cos 0 + \frac{4}{(1+w^2)^2} - \frac{2}{(1+w^2)^2} = 0$

$= 0 + wD_2 - \frac{2}{(1+w^2)^2} + 2w \Rightarrow D_2 = \frac{(\frac{2}{(1+w^2)^2} - 2w^2)/w}{w}$

$= 0 + wD_2 - \frac{2}{(1+w^2)^2} + 2w \Rightarrow D_2 = \frac{2(1-w^2)}{(1+w^2)^2 \cdot w}$

$y = \frac{1}{(1+w^2)^2} \cdot (-4 \cos wt + \frac{2-2w^2}{w} \sin wt + 4e^{-t} + 2te^{-t} + 2w^2 te^{-t})$

for $t \rightarrow \infty \quad A = \sqrt{16 + \frac{(2-2w)^2}{w^2}}$

$\frac{-2(w-1)}{(w^2+1)^2}$

13X

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14Y

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18Y

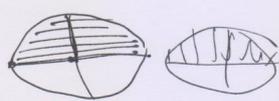
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Solution(s):

From user: ar857

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a) $\int_0^1 \int_0^2 x e^{xy} dx dy = \int_0^2 e^{xy} \Big|_0^1 dx = \int_0^2 (e^x - 1) dx = e^2 - 2 - 1 = e^2 - 3$

b) $\int_0^{1/2} \int_{-\sqrt{1-4y^2}}^{\sqrt{1-4y^2}} y dx dy$ $x^2 = 1 - 4y^2$
 $y^2 = \frac{1-x^2}{4}$ 

$= \int_{-1}^1 \int_0^{\sqrt{\frac{1-x^2}{4}}} y dy dx = \int_{-1}^1 \frac{1}{8} (1-x^2) dx = \frac{1}{8} \cdot \left(1 - \frac{1}{3} - \frac{1}{3} + 1\right) = \frac{1}{8} \cdot \frac{4}{3} = \frac{1}{6}$

c) $0 \leq r \leq 1$ $0 \leq \theta \leq \frac{1}{2}\pi$ $x+y+z=4$ $z=0 \Rightarrow 4-x-y \leq z \leq 0$

$\iiint_V dx dy dz = \int_0^{\pi/2} \int_0^1 \int_0^{4-r(\cos\theta+\sin\theta)} r dz dr d\theta$

$= \int_0^{\pi/2} \int_0^1 4r - r^2(\cos\theta + \sin\theta) dr d\theta = \int_0^{\pi/2} 2 - \frac{\cos\theta + \sin\theta}{3} d\theta = 2 \cdot \frac{\pi}{2} - \frac{1}{3}(1+0-0+1) = \pi - \frac{2}{3}$

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19R*

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