2008 Mathematics (1)

Section A

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Section B

11Y

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Solution(s):

From user: cgl20

$K\underline{r} = \underline{r} - (\underline{n}.\underline{r})\underline{n}$, $|\underline{n}| = 1$.

$$K^2\underline{r} = (\underline{r} - (\underline{n}.\underline{r})\underline{n}) - (\underline{n}.(\underline{r} - (\underline{n}.\underline{r})\underline{n}))\underline{n}$$

$$= \underbrace{\underline{r} - (\underline{n}.\underline{r})\underline{n}} - \cancel{(\underline{n}.\underline{r})\underline{n}} + \underbrace{|\underline{n}|^2}_{=1}\cancel{(\underline{n}.\underline{r})\underline{n}}$$

$$= K\underline{r} \qquad \underline{\underline{\text{QED.}}}$$

Suppose $\underline{a} /\!/ \underline{n}$, ie $\underline{a} = \lambda \underline{n}$.

Then $K\underline{a} = \lambda\underline{n} - (\underline{n}.\lambda\underline{n})\underline{n}$

$= \lambda\underline{n} - \lambda\underline{n}$

$= \underline{0}$ $\underline{\underline{\text{QED}}}$

Suppose $\underline{b}.\underline{n} = 0$.

Then $K\underline{b} = \underline{b} - (\overset{0}{\cancel{\underline{n}.\underline{b}}})\underline{n} = \underline{b}$ $\underline{\underline{\text{QED.}}}$

Any vector $\underline{r}$ can be decomposed into parts parallel & perp to $\underline{n}$:

$\underline{r} = \underline{r}_{/\!/} + \underline{r}_{\perp}$ where $\underline{r}_{/\!/} = \lambda\underline{n}$, $\underline{r}_{\perp}.\underline{n} = 0$.

$\therefore K\underline{r} = K\underline{r}_{/\!/} + K\underline{r}_{\perp}$ (by linearity of K)

$= \underline{0} + \underline{r}_{\perp}$ (by last two results)

$= \underline{r}_{\perp}$ $\underline{\underline{\text{QED.}}}$

Now let $\underline{n} = (1,1,1)/\sqrt{3}$

$\underline{e}_1$ & $\underline{e}_2$ satisfy $\underline{e}_1.\underline{n} = \underline{e}_2.\underline{n} = 0$.

$\underline{e}_1 \propto (1, 1, ?)$ (since equally inclined to x & y axes)

$\therefore \underline{e}_1.\underline{n} = 0 \Rightarrow \underline{e}_1 = (1, 1, -2)/\sqrt{6}$.

$\therefore \underline{e}_2 =$ (by inspection) $= (1, -1, 0)/\sqrt{2}$.

Suppose:
$\underline{r} = \lambda\underline{n} + \mu_1\underline{e}_1 + \mu_2\underline{e}_2$. $\underline{r} = (x, y, z)$

Since $\underline{n}, \underline{e}_1, \underline{e}_2$ orthogonal, can find λ, μ_1, μ_2 by dotting

$\Rightarrow \mu_1 = \underline{r}.\underline{e}_1 = (x+y-2z)/\sqrt{6}$. (ignoring centre translation)

$\mu_2 = \underline{r}.\underline{e}_2 = (x-y)/\sqrt{2}$.

Circles in plane $z=0$ could be characterised by being points for which $x^2+y^2 = r^2$ (fixed) and $z=0$.

For these $\left\{ \begin{array}{l} \mu_1 = (x+y)/\sqrt{6} \\ \mu_2 = (x-y)/\sqrt{2} \end{array} \right\}$ from above.

Evidently, major & minor axes are @ 45° to $\underline{e}_1$ & $\underline{e}_2$, and the ratio of their lengths is $\sqrt{6}/\sqrt{2} = \sqrt{3}$.

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13Z

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Solution(s):

From user: ar857

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a) $f(x) = \ln(1+x) \qquad f(0) = 0$

$f'(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 \qquad f'(0) = 1$

$f''(x) = -1(1+x)^{-2} \qquad f''(0) = -1$

$f'''(x) = 2(1+x)^{-3} \qquad f'''(0) = 2$

$f^{IV}(x) = -6(1+x)^{-4} \qquad f^{IV}(0) = -6$

$f^{V}(x) = 24(1+x)^{-5} \qquad f^{V}(0) = 24$

$\therefore$

$$\boxed{\begin{aligned} \ln(1+x) &= 0 + x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{6x^4}{4!} + \frac{24x^5}{5!} \dots \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \dots + (-1)^{n+1}\frac{x^n}{n} + \dots \end{aligned}}$$

$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \dots$

$\ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} \dots + \frac{x^n}{n} + \dots\right)$

$\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x) = x + x - \frac{x^2}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^4}{4} \dots$

$= 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots + \frac{x^n}{n} \dots\right)$

b) $(1-e^x)\cdot\left(1+\frac{x}{3}\right)^{-3} + \ln(1-x)$

$(1-e^x) = 1 - \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \dots \frac{x^n}{n!}\right) = -x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{24}$

$\left(1+\frac{x}{3}\right)^{-3} = 1 - x + \frac{+12}{2}\cdot\frac{x^2}{9} - \frac{60}{6}\cdot\frac{x^3}{27} + \frac{360}{24}\,\frac{x^4}{81}$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots$

$(1-e^x)\cdot\left(1+\frac{x}{3}\right)^{-3} + \ln(1-x) = \left(-x + x^2 - \frac{x^2}{2} - \frac{2}{3}x^3 + \frac{x^3}{2} - \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^4}{6} - \frac{x^4}{3} + \frac{10}{27}x^4\right)$

$+ x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

$= 0 + 0 + 0 + x^4\cdot\left(\frac{10}{27} - \frac{1}{3} + \frac{1}{6} - \frac{1}{24} - \frac{1}{4}\right) = x^4\cdot\left(\frac{1}{27} - \frac{1}{8}\right)$

$= x\frac{8-27}{216} = \frac{-19}{216}x^4$

c) $(1+ax)(1+bx)^{-1}\cdot\ln(1+x)$

$= (1+ax)\cdot(1 - bx + b^2x^2 - b^3x^3)\cdot\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right)$

$= (1 - bx + b^2x^2 - b^3x^3 + ax - abx^2 + ab^2x^3 - ab^3x^4)\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right)$

$0 = x^2\cdot\left(-\frac{1}{2} - b + a\right) \Rightarrow a = b + \frac{1}{2}$

$0 = x^3\cdot\left(\frac{1}{3} + \frac{b}{2} + b^2 - \frac{a}{2} - ab\right) = \frac{1}{3} + \frac{b}{2} + b^2 - \frac{b}{2} - \frac{1}{4} - b^2 - \frac{b}{2} = \frac{1}{12} - \frac{b}{2} = 0$

$\Rightarrow b = \frac{1}{6} \qquad a = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$

$x^4\cdot\left(-\frac{1}{4} - \frac{b}{3} - \frac{b^2}{2} - b^3 + \frac{a}{3} + \frac{ab}{2} + ab^2\right) = x^4\cdot\left(-\frac{1}{4} - \frac{1}{18} - \frac{1}{72} - \frac{1}{216} + \frac{2}{9} + \frac{1}{18} + \frac{1}{54}\right)$

$= x^4\cdot\left(\frac{-9 - 2 - 0.5 - \frac{1}{6} + 8 + 2 + \frac{2}{3}}{36}\right) = \frac{-1}{36}$

14R

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15X

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16Z

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17T

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Solution(s):

From user: ar857

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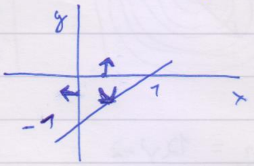
a) i) $f(x,y) = xy(1-x+y)$

$f=0$ at $x=0$ at $y=0$ and $y=x-1$



ii) $\frac{\partial f}{\partial x} = y(1-x+y) - xy$

$\frac{\partial f}{\partial y} = x(1-x+y)+xy$

∇f at $(\frac{1}{2},0) = (0, \frac{1}{4})$

$(\frac{1}{2},-\frac{1}{2}) = (\frac{1}{4}, -\frac{1}{4})$

$(0,-\frac{1}{2}) = (-\frac{1}{4}, 0)$

iii) $\frac{\partial f}{\partial x} = 0$ $y \cdot (1-2x+y)$ $x \cdot (1-x+2y)$

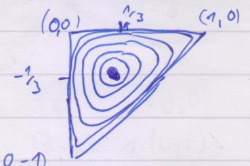
for $y=0$ or $y=2x-1$

$\frac{\partial f}{\partial y} = 0$ for $x=0$ or $x = 2y+1$

within the triangle

$y = 4y+2-1$ $y = -\frac{1}{3}$ $x = \frac{1}{3}$ is $(\frac{1}{3},-\frac{1}{3})$

$f(\frac{1}{3},-\frac{1}{3}) = -\frac{1}{9} \cdot (\frac{1}{3}) = -\frac{1}{27}$ minimum



b) $g = y^2(1-x) - x^2(1-x)$

$\frac{\partial g}{\partial x} = -y^2 - 2x - 3x^2$

$\frac{\partial g}{\partial y} = 2y - 2yx$ $= 0$ for $y=0$ or $x=1$

$\frac{\partial g}{\partial x}(x,0) = -2x-3x^2 = -x(2+3x)$

$= 0$ for $x=0$ and $x = -\frac{2}{3}$

$\frac{\partial g}{\partial x}(1,0) = -y^2 - 5 \neq 0$

$(0,0)$; $(0,-\frac{2}{3})$

$f_{xx} = -2-6x$ $(0,0)$ is saddle

$f_{xy} = -2y$ $(0,-\frac{2}{3})$ is a minimum

$f_{yy} = 2-2x$

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19X*

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20Y*

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