

2008 Mathematics (2)

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Section A

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Section B

10Y

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11X

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12Y

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13Z

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Solution(s):

From user: ar857

2068
2018 II 137

a) $y'' - 7y' + 12y = 144x$
 $\lambda^2 - 7\lambda + 12$

$$y_c = c_1 e^{4x} + c_2 e^{3x}$$

$$y_p = Kx + B$$

$$y = 0 - 7K + 12Kx + 12B = 144x \Rightarrow K=12 \quad B=7$$

$$y = c_1 e^{4x} + c_2 e^{3x} + 12x + 7$$

$$y(0) = c_1 + c_2 + 7 = 0$$

$$y'(0) = 4c_1 + 3c_2 + 12 = 0 \quad c_1 - 2c_2 + 12 = 0 \quad c_2 = -16 \quad c_1 = 9$$

$$y = 9e^{4x} - 16e^{3x} + 12x + 7$$

b) $y'' + 3y' + 2y = 10\sin x$
 $\lambda^2 + 3\lambda + 2 = 0 \quad \lambda = -2 \quad \lambda = -1$

$$y_p = K \sin x + B \cos x$$

$$y'_p = K \cos x - B \sin x$$

$$y''_p = -K \sin x - B \cos x$$

$$y_g = -K \sin x - B \cos x + 3K \cos x - 3B \sin x + 2K \sin x + 2B \cos x = 10 \sin x$$

$$-3B + 2K = 10$$

$$B + 3K + 2B = 0 \quad B = -3 \quad K = 1$$

$$y = c_1 e^{-2x} + c_2 e^{-x} + \sin x - 3 \cos x$$

$$y(0) = c_1 + c_2 - 3 = 0$$

$$y'(0) = -2c_1 - c_2 + 1 = 0 \quad -6 + 2c_2 - c_2 + 1 = 0 \quad c_2 = 5 \quad c_1 = -2$$

$$y = -2e^{-2x} + 5e^{-x} + \sin x - 3 \cos x$$

c) $y'' + 2y' + y = 8e^{-x}$

$$\lambda^2 + 2\lambda + 1 \quad y_c = c_1 e^{-x} + c_2 x e^{-x}$$

$$y_p = Kx^2 e^{-x}$$

$$y_g = 2e^{-x} - 2x e^{-x} + 2(2x e^{-x} K - x^2 e^{-x} K) + x^2 e^{-x} K = 8e^{-x}$$

$$y_g = 2e^{-x} K = 8e^{-x} \quad K = 4$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + 4x^2 e^{-x}$$

$$y(0) = c_1 + c_2 = 0$$

$$y'(0) = -c_1 + c_2 = 0$$

$$y = 4x^2 e^{-x}$$

14T

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Solution(s):

From user: ar857

2008 14 T II

$$\cosh^2 - \sinh^2 = 1$$

a) $U = A \cdot \cosh^{-2}(x-vt)$

$$\partial U / \partial t = -2A \cosh^{-3}(x-vt) \sinh(x-vt)$$

$$\partial U / \partial x = -2A \cosh^{-3} \sinh$$

$$\partial^3 U / \partial x^3 = -2A \cosh^{-3} + 6A \cosh^{-4} \sinh^2$$

$$\partial^3 U / \partial x^3 = 4A \cosh^{-3} \sinh + 12A \cosh^{-4} \sinh \cosh - 24A \cosh^{-5} \sinh^3$$

$$\frac{\partial U}{\partial t} - 6U \frac{\partial U}{\partial x} + \frac{\partial^3 U}{\partial x^3} = \frac{24V \sinh}{\cosh^3} + 12 \frac{A \sinh}{\cosh^3} \cdot \frac{A}{\cosh^2} + \frac{4A \sinh}{\cosh^3} + \frac{12A \sinh}{\cosh^3}$$

$$= \frac{24A \sinh (\cosh^2 - 1)}{\cosh^5}$$

$$= \frac{\sinh}{\cosh^5} \cdot (-4V - 24 - 8 + 48) + \frac{\sinh}{\cosh^5} (24A + 12A^2) = 0$$

for $A = -2$ $V = 4$

b) $\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial t} \right) + \left(\frac{\partial f}{\partial x} \right) \left(\frac{\partial f}{\partial x} \right) = k \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial t} \left(\frac{\partial f}{\partial x} \right) + \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) = k \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right)$

$$= \frac{\partial g}{\partial t} + g \frac{\partial g}{\partial x} = k \frac{\partial^2 g}{\partial x^2} \quad \checkmark$$

ii) $\frac{\partial g}{\partial t} = -\frac{1}{2k} \frac{\partial}{\partial t} e^{-t/2k}$

$$\frac{\partial g}{\partial x} = -\frac{1}{2k} \frac{\partial}{\partial x} e^{-t/2k}$$

$$k \frac{\partial^2 g}{\partial x^2} = k \left(-\frac{1}{2k} \frac{\partial^2}{\partial x^2} e^{-t/2k} + \frac{1}{4k^2} \left(\frac{\partial f}{\partial x} \right)^2 e^{-t/2k} \right)$$

$$= e^{-t/2k} \cdot \frac{1}{2k} \left(\frac{1}{2} \left(\frac{\partial f}{\partial x} \right)^2 - k \frac{\partial^2 f}{\partial x^2} \right) = -e^{-t/2k} \frac{1}{2k} \frac{\partial}{\partial t} e^{-t/2k} = \frac{\partial g}{\partial t} \quad \checkmark$$

15Z

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16S

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17X

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18R*

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19T*

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