

2007 Mathematics (1)

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Section A

1

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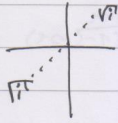
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Solution(s):

From user: ar857

2007 I 2T

a) i) $\sqrt{1}$



$$z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

$$z_2 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

ii) $\ln(-e) = \ln(e) + \ln(-1) = 1 + \ln(\cos \pi + i \sin \pi) = 1 + \ln e^{i\pi} = 1 + i\pi$

iii) $2^i = z$

$$\ln z = i \ln 2 \quad e^{\ln z} = z = e^{i \ln 2} = \cos(\ln 2) + i \sin(\ln 2)$$

iv) $(i)^i = i^{i^2} = i^{-1} = \frac{1}{i} = \frac{1}{i} \cdot \frac{-i}{-i} = -i$

b) $\sin 3x = \operatorname{Im}[e^{ix}]^3 = \operatorname{Im}[(\cos x + i \sin x)^3]$

$$= 3 \cos^2 x \sin x - \sin^3 x = 3 \sin x - 4 \sin^3 x$$

c) $3 \sin x - 4 \sin^3 x = 2 \sin x$

$$\sin x = \sin x + 4 \sin^3 x$$

Solution #1: $\sin x = 0 \therefore x = 0 + k\pi$

Solution #2: $\sin x = \pm \frac{1}{2} \therefore x = \frac{\pi}{6} + \frac{2}{3}k\pi$
or $x = \frac{7\pi}{6} + \frac{2}{3}k\pi$

3

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Solution(s):

From user: ar857

Handwritten solutions for two differential equations:

Q1) (1+e^x) cos x dx + e^x sin x dy = 0 is exact
 $e^x \sin x + \sin x = D$

ii) $y(x^2 + \ln y) dx + x dy = 0$
 $1 - 1 - \ln y - x^2$
 $-y \cdot (x^2 + \ln y) dy = \frac{1}{x} dx$
 $-\frac{1}{y} dy = \frac{1}{x} dx$
 $\mu = \frac{1}{y}$
 $\frac{1}{3} x^3 + x \ln y = C$

b) $\frac{dy}{dx} - \frac{1}{(1+x)} 3y = (1+x)^4$
 $(x+1)^{-3} y' = \int x+1 = \frac{3}{2} x^2 + x + C$
 $y = \frac{1}{(1+x)^3} \cdot \frac{1}{2} \cdot (x^2 + 2x + 3)$
 $= \frac{1}{(1+x)^3} \cdot \frac{1}{2} \cdot [(1+x)^2 + 2] = \frac{(1+x)^2}{2} + (1+x)^3$

6

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Solution(s):

From user: ar857

2007 I B6

$$\theta_1 = x \quad \theta_2 = y$$

$$U = -3 \cos(x+y) - \cos(x-y)$$

$$\frac{\partial U}{\partial x} = 3 \sin(x+y) + \sin(x-y)$$

$$\frac{\partial^2 U}{\partial x^2} = 3 \cos(x+y) + \cos(x-y)$$

$$\frac{\partial U}{\partial y} = 3 \sin(x+y) - \sin(x-y)$$

$$\frac{\partial^2 U}{\partial y^2} = 3 \cos(x+y) - \cos(x-y)$$

$$\cos(x+y) = 0 \quad \text{and} \quad \cos(x-y) = 0$$

$$x+y = 0$$

$$x-y = 0$$

$$x = y$$

$$x+y = \pi$$

$$x-y = \pi$$

$$x = \pi + y$$

$$x+y = 2\pi$$

$$x-y = -\pi$$

$$x = -\pi + y$$

$$x \neq y \neq \pi$$

$$\begin{aligned} U_{xx} &= 4 \\ U_{yy} &= 4 \\ U_{xy} &= 2 \end{aligned}$$

$$(0,0)$$

$$(\pi/2, \pi/2)$$

$$(\pi, \pi)$$

$$(\pi, 0)$$

$$(0, \pi)$$

$$\begin{aligned} U_{xx} &= -4 \\ U_{yy} &= -4 \\ U_{xy} &= -2 \end{aligned}$$

min

saddle

min

max

max

$$a = \left(\frac{\pi}{2}, \frac{\pi}{2} \right) \quad -3 + 1 = -2 \Rightarrow U_{xx} = U_{yy}$$

$$U_{xy} = -4$$

$$a = \left(\frac{\pi}{2}, \frac{\pi}{2} \right) \quad U_{xx} U_{yy} = 4 < (U_{xy})^2 = 16 \Rightarrow$$

\hookrightarrow Saddle

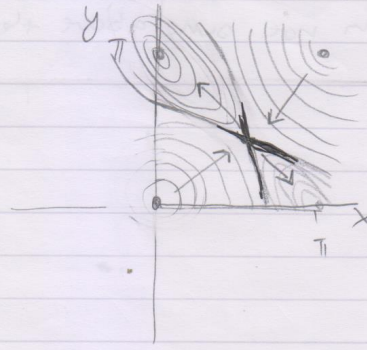
$$\frac{dy}{dx} = \frac{1}{-2} (4 \pm \sqrt{16-4})$$

$$\frac{dy}{dx} = -2 \pm \sqrt{3} = 0.3 \sim -3.7$$

$$\sim 0.3 \sim -3.7$$

$$\underline{\underline{-2.547}}$$

$$-2\sqrt{3}$$



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Section B

1Y

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Solution(s):

From user: cgl20

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Rosalind's supervision test

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3Z

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4X

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5Z

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6T

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7R

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8S

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9X*

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10Y*

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