2007 Mathematics (2)

Section A

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Solution(s):

From user: ar857

2007 II 4

a) $y'' + 4y' + 4y = e^{2x}$

$y = Ae^{-2x} + Bxe^{-2x}$

$y_p = ke^{2x} \quad = \frac{1}{16}e^{2x}$

$Ly = 4k + 8k + 4k = 1 \quad k = \frac{1}{16}$

$y = (A + Bx)e^{-2x} + \frac{1}{16}e^{2x}$

b) $y'' + 6y' + 25y = 30\cos 5x$

$\lambda^2 + 6\lambda + 25 = 0 \quad \lambda = \frac{-6 \pm \sqrt{36-100}}{2} = -3 \pm 4i$

$y_c = e^{-3x} \cdot (D_1 \cos 4x + D_2 \sin 4x)$

$y_p = K\cos 5x + B\sin 5x$

$y_p = -K\,25\cos 5x - 25B\sin 5x + 30B\cos 5x - 30k\sin 5x = 30\cos 5x$

$+ 25k\cos 5x + 25B\ldots$ $\quad k = 0,\ B = 1$

$y = e^{-3x}(D_1\cos 4x + D_2\sin 4x) + \sin 5x$

$D_1 = 0$

~~$y' = -3e^{-3x}(\ldots) \ldots = 0$~~

$e^{-3x} D_2 \sin 4x + \sin 5x$

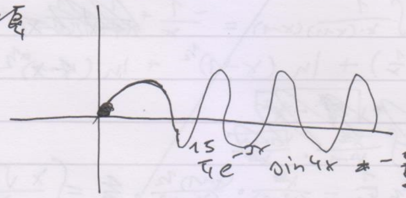
$y'(0) = 0 \quad 4D_2 + 5 = 0 \quad D_2 = -\frac{5}{4}$

$y = -\frac{5}{4}e^{-3x}\sin 4x + \sin 5x$

c)

$x \to \infty \quad \to 0$

$x \to 0 \quad \to 0$



$\frac{15}{4}e^{-3x}\sin 4x - \frac{5}{?}e^{-3x}\sin 4x - 5\cos 5x$

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Solution(s):

From user: ar857

2007 II B5

$U(x,y)dx + V(x,y)dy$

a) is exact only if $\left(\frac{\partial U}{\partial y}\right)_x = \left(\frac{\partial V}{\partial x}\right)_y$ $T = \left(\frac{\partial U}{\partial S}\right)_V$

b) $\left(\frac{\partial V}{\partial U}\right)_y \left(\frac{\partial U}{\partial x}\right)_y = \left(\frac{\partial V}{\partial x}\right)_y$ $-p = \left(\frac{\partial U}{\partial V}\right)_S$

$dU = TdS - pdV$

$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V = -\left(\frac{\partial p}{\partial U}\right)_V \left(\frac{\partial U}{\partial S}\right)_V = -\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial U}\right)_V T$

$= -\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V$

c) $pV = RT$

$\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{\partial}{\partial V}\left(\frac{pV}{R}\right)\right)_S = \left(\frac{\partial}{\partial V}\left(\frac{kV^{-\frac{2}{3}}}{R}\right)\right)_S = -\frac{2}{3}\frac{kV^{-\frac{5}{3}}}{R} = -\frac{2}{3}\frac{p}{R} = -\frac{2}{3}\frac{T}{V}$

$-\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V = -\frac{2}{3}\frac{T}{R}\left(\frac{\partial}{\partial T}\left(\frac{RT}{V}\right)\right)_V = -\frac{2}{3}\frac{T}{R}\frac{R}{V} = -\frac{2}{3}\frac{T}{V}$

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Solution(s):

From user: ar857

2007 II B6 ✓

a) $x = b\sin\theta\cos\phi,\ b\sin\theta\sin\phi,\ a\cos\theta$

$b\cos\theta\cos\phi,\ b\cos\theta\sin\phi,\ -a\sin\theta$

$-b\sin\theta\sin\phi,\ b\sin\theta\cos\phi,\ 0$

$A = (ab\sin\theta^2\cos\phi,\ ab\sin\theta^2\sin\phi,\ b^2\cos\theta\sin\theta)\,d\theta\,d\phi$

$G = (ba^2\sin\theta\cos\phi\cos\theta^2,\ b^3\sin^3\theta\cos\phi\sin\phi,\ a^3\cos\theta^3)$

$G\cdot dS = b^2a^3\sin\theta^3\cos\theta^2\cos\phi^2 + b^4a\sin\theta^5\cos\phi\sin\phi^2,\ b^2a^3\cos\theta^4\sin\theta$

$\int G\cdot dS \int_0 \left[\frac{b^2a^3\sin\theta^3\cos\theta^2}{2}x + \frac{b^2a^3\sin\theta^3\cos\theta^2}{4}\sin(2x) / \frac{b^2a^3\cos^4}{\sin\theta}x\right]_0^{2\pi}$

$\int_0^{\pi} b^2a^3\pi\ (\sin\theta^3\cos\theta^2 + 2\cos\theta^4\sin\theta)$

$b^2a^3\pi \int_0^{\pi} \sin\cos\theta^2 + \sin\theta\cos\theta^4 -$

$= b^2a^3\pi\ \left[-\frac{1}{3}\cos\theta^3 + -\frac{1}{5}\cos\theta^5\right]_0^{\pi}$

$= b^2a^3\pi\ \ \frac{1}{3} + \frac{1}{5} + \frac{1}{3} + \frac{1}{3} = \frac{16}{15}b^2a^3\pi$

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Section B

1Y

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2Y

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3X

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4Z

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5T

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6Z

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7S

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8X

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9R*

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10T*

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