

2007 Mathematics (2)

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Section A

1

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Solution(s):

From user: ar857

2007 II 4

a) $y'' + 4y' + 4y = e^{2x}$

$$y = k e^{2x} + c_1 x e^{2x}$$

$$y_p = k e^{2x} = \frac{1}{16} e^{2x}$$

$$4k = 4k^2 + 8k + 4k = 1 \quad k = \frac{1}{16}$$

$$y = (A + Bx) e^{-2x} + \frac{1}{16} e^{2x}$$

b) $y'' + 6y' + 25y = 30 \cos 5x$

$$\lambda^2 + 6\lambda + 25 = 0 \quad \lambda = \frac{-6 \pm \sqrt{36 - 100}}{2} = -3 \pm 4i$$

$$y_c = e^{-3x} \cdot (D_1 \cos 4x + D_2 \sin 4x)$$

$$y_p = K \cos 5x + B \sin 5x$$

$$y_p = K 25 \cos 5x - 25B \sin 5x + 30B \cos 5x - 30K \sin 5x = 30 \cos 5x$$

$$y = e^{-3x} (D_1 \cos 4x + D_2 \sin 4x) + 5 \sin 5x$$

$$D_1 = 0$$

$$y = e^{-3x} D_2 \sin 4x + 5 \sin 5x$$

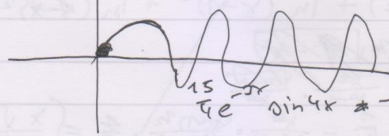
$$e^{-3x} D_2 \sin 4x + 5 \sin 5x$$

$$y'(0) = 0 \quad 4D_2 + 5 = 0 \quad D_2 = -\frac{5}{4}$$

$$y = -\frac{5}{4} e^{-3x} \sin 4x + 5 \sin 5x$$

c) $\frac{1}{4}$

$$\begin{matrix} x \rightarrow \infty & \rightarrow 0 \\ x \rightarrow 0 & = 0 \end{matrix}$$



$$\frac{1}{4} e^{-3x} \sin 4x + 5 \sin 5x$$

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Solution(s):

From user: ar857

2007 II B5

a) is exact if $\left(\frac{\partial u}{\partial y}\right)_x = \left(\frac{\partial v}{\partial x}\right)_y$

$$T = \left(\frac{\partial U}{\partial S}\right)_V$$

b) $\left(\frac{\partial v}{\partial u}\right)_y \left(\frac{\partial u}{\partial x}\right)_y = \left(\frac{\partial v}{\partial x}\right)_y$

$$-P = \left(\frac{\partial U}{\partial V}\right)_S$$

$$dU = TdS - PdV$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V = -\left(\frac{\partial P}{\partial U}\right)_V \left(\frac{\partial U}{\partial S}\right)_V = -\left(\frac{\partial P}{\partial T}\right)_V T$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{T}{C_V} \left(\frac{\partial P}{\partial T}\right)_V$$

c) $PV = RT$

$$\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{\partial}{\partial V} \left(\frac{PV}{R}\right)\right)_S = \left(\frac{\partial}{\partial V} \left(\frac{KV^{-\frac{2}{3}}}{R}\right)\right)_S = -\frac{2}{3} \frac{KV^{-\frac{5}{3}}}{R} = -\frac{2}{3} \frac{P}{R} = -\frac{2}{3} \frac{T}{V}$$

$$-\frac{T}{C_V} \left(\frac{\partial P}{\partial T}\right)_V = -\frac{2}{3} \frac{T}{R} \left(\frac{\partial}{\partial T} \left(\frac{RT}{V}\right)\right)_V = -\frac{2}{3} \frac{T}{R} \frac{R}{V} = -\frac{2}{3} \frac{T}{V}$$

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Solution(s):

From user: ar857

2007 II B6 ✓

a) $x = b \sin \sigma \cos \phi, y = b \sin \sigma \sin \phi, z = a \cos \sigma$
 $dx = b \cos \sigma \cos \phi, dy = b \cos \sigma \sin \phi, dz = -a \sin \sigma$
 $d\sigma = -b \sin \sigma \sin \phi, d\phi = b \sin \sigma \cos \phi, 0$

$$dV = (ab \sin^2 \sigma \cos \phi, ab \sin^2 \sigma \sin \phi, b^2 \cos \sigma \sin \sigma) d\sigma d\phi$$

$$G = (b^2 a^2 \sin \sigma \cos \phi \cos^2 \sigma, b^3 \sin^3 \sigma \cos \phi \sin \phi, a^3 \cos^3 \sigma)$$

$$G \cdot dV = b^2 a^3 \sin^3 \sigma \cos^3 \phi \cos^2 \sigma + b^4 a \sin^5 \sigma \cos \phi \sin^2 \phi + b^3 a^3 \cos^3 \sigma \sin \sigma$$

$$\int_0^\pi \int_0^{2\pi} \left[\frac{b^2 a^3 \sin^3 \sigma \cos^2 \phi}{2} x + \frac{b^2 a^2 \sin^3 \sigma \cos \phi}{4} \sin(2\phi) - \frac{b^2 a^3 \cos^3 \sigma}{\cos^2 \sigma \sin \sigma} x \right]_0^{2\pi} d\sigma$$

$$\int_0^\pi b^2 a^3 \pi (\sin^3 \sigma \cos^2 \sigma + 2 \cos^4 \sigma \sin \sigma) d\sigma$$

$$b^2 a^3 \pi \int_0^\pi \sin \cos^2 \sigma + \sin \sigma \cos^4 \sigma d\sigma$$

$$= b^2 a^3 \pi \left[-\frac{1}{3} \cos^3 \sigma + \frac{1}{5} \cos^5 \sigma \right]_0^\pi$$

$$= b^2 a^3 \pi \left(\frac{2}{3} + \frac{2}{5} + \frac{1}{5} + \frac{1}{3} \right) = \frac{16}{15} b^2 a^3 \pi$$

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Section B

1Y

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2Y

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3X

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4Z

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5T

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6Z

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7S

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8X

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9R*

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10T*

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