

2005 Mathematics (1)

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1A

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2A*

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3B

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5C

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6C*

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7D*

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8D

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9E

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Solution(s):

From user: ar857

2005 I QE

a) $y' = \frac{y^{2+1}}{\cos^2 x} \Rightarrow \frac{1}{y^{2+1}} dy = \frac{1}{\cos^2 x} dx$
 $\arctan y = \tan x + c$
 $y = \tan(\tan x + c)$

b) $y' + \frac{x+\alpha}{x} y = \frac{e^{-x}}{x}$ $e^{\int \frac{x+\alpha}{x} dx} = e^{\int 1 + \frac{\alpha}{x} dx} = e^{x+\alpha \ln x} = e^x \cdot x^\alpha$
 $e^x x^\alpha y = \int x^{\alpha-1} = \frac{1}{\alpha} x^\alpha + c$
 $y = \frac{1}{\alpha} e^{-x} + \frac{c}{x^\alpha} e^{-x}$
 if $\alpha=0$ $e^x y = \ln x + c$ $y = \ln x e^{-x} + c e^{-x}$

c) $y' + 4xy = 2xy^2 + 2x$
 $y' = x(2y^2 + 2 - 4y) = 2x(y-1)^2$
 $\frac{dy}{(y-1)^2} = 2x dx$
 $-\frac{1}{y-1} = x^2 + c$
 $y = \frac{1}{-x^2 - c} + 1$

10E

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Solution(s):

From user: ar857

2005 I 10

$$y'' - (a+b)y' + aby = e^{cx}$$

a) $a \neq b \neq c \neq a$

$$\lambda^2 - (a+b)\lambda + ab = 0 \Rightarrow \lambda_1 = a, \lambda_2 = b$$

$$y_c = c_1 e^{ax} + c_2 e^{bx}$$

$$y_p = K e^{cx} \quad L y_p = K e^{cx} (c^2 - (a+b)c + ab) = e^{cx}$$

$$\Rightarrow K = (c^2 - (a+b)c + ab)^{-1} = (c^2 - ac - bc + ab) = [(c-a)(c-b)]^{-1}$$

$$y(0) = c_1 + c_2 + K = 0 \Rightarrow c_1(c-a)(c-b) + c_2(c-a)(c-b) + 1 = 0$$

$$y'(0) = ac_1 + bc_2 + \frac{c}{(c-a)(c-b)} = 0$$

$$c_1 = \frac{-1}{(c-a)(c-b)} - c_2$$

$$\frac{-a}{(c-a)(c-b)} + (b-a)c_2 + \frac{c}{(c-a)(c-b)} = 0 \Rightarrow \frac{1}{(c-b)} + \frac{(b-a)}{(c-a)} c_2 = 0 \Rightarrow c_2 = \frac{1}{(a-b)(c-b)}$$

$$c_1 = \frac{1}{(c-a)(b-a)}$$

$$y = \frac{1}{(c-a)(b-a)} e^{ax} + \frac{1}{(c-b)(a-b)} e^{bx} + \frac{1}{(c-b)(c-b)} e^{cx}$$

b) $a \neq b = c$

$$y'' - (a+b)y' + aby = e^{bx} \quad y_c = c_1 e^{ax} + c_2 e^{bx}$$

$$y_p = K x e^{bx}$$

$$L y_p = (2bK e^{bx} + K e^{bx} + bK x e^{bx}) (-a-b) + abK x e^{bx} = (b-a) K e^{bx} = e^{bx} \Rightarrow K = \frac{1}{b-a}$$

$$y = c_1 e^{ax} + c_2 e^{bx} + \frac{1}{b-a} x e^{bx}$$

$$y(0) = c_1 + c_2 = 0 \Rightarrow c_1 = -c_2$$

$$y'(0) = ac_1 + bc_2 + \frac{1}{b-a} = 0 \Rightarrow (a-b)c_1 = \frac{1}{b-a} \quad c_1 = \frac{1}{(a-b)^2} \quad c_2 = \frac{1}{(a-b)^2}$$

$$y = \frac{1}{(a-b)^2} e^{ax} - \frac{1}{(a-b)^2} e^{bx} + \frac{1}{(b-a)} x e^{bx}$$

11F

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Solution(s):

From user: ar857

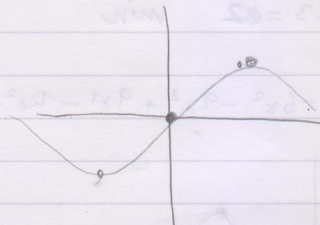
2005 I 11

a) $f = (x^2 + 1)e^{-x^2}$

$f' = (1 + 3x^2 - 2x^2 - 2x^4)e^{-x^2} = -(2x^4 - x^2 - 1)e^{-x^2} = e^{-x^2}(x^2 - 1)(2x^2 + 1)$

$\max_{x=1} \quad \min_{x=-1}$

b) $f'' = -e^{-x^2}(8x^3 - 4x^5 + 2x^3) = -\frac{1}{2}e^{-x^2}(5 - 2x^2)$



$x^2 \cdot x e^{-x^2}$

c) i) = 0

ii) $\int_0^\infty x e^{-x^2} + x^3 e^{-x^2} = -\frac{1}{2}e^{-x^2} + x^2 \cdot -\frac{1}{2}e^{-x^2} + \frac{1}{2}e^{-x^2} = -\left(\frac{x^2}{2} + 1\right)e^{-x^2} \Big|_0^\infty = 1$

iii) $\int_0^\infty f(x) \frac{d f(x)}{d x} d x = f(x) \circ f(x) - \int_0^\infty f(x) \frac{d f}{d x}$

$= \frac{1}{2} \left((x+1)^2 e^{-x^2} - (x-1)^2 e^{-x^2} \right) \Big|_0^\infty = \frac{1}{2} \cdot (0 - 0) = 0$

12F

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