2005 Mathematics (1)

1A

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2A*

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3B

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5C

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6C*

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7D*

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8D

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9E

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Solution(s):

From user: ar857

2005 I 9E

a) $y' = \frac{y^2+1}{\cos^2 x} \Rightarrow \frac{1}{y^2+1}\,dy = \frac{1}{\cos^2 x}\,dx$

$$\arctan y = \tan x + c$$

$$y = \tan(\tan x + c)$$

b) $y' + \frac{x+\alpha}{x}\,y = \frac{e^{-x}}{x}$ $\quad e^{\int \frac{x+\alpha}{x}} = e^{\int 1+\frac{\alpha}{x}} = e^{x+\alpha\ln x} = e^x \cdot x^\alpha$

$$e^x x^\alpha y = \int x^{\alpha-1} = \frac{1}{\alpha}x^\alpha + c$$

$$y = \frac{1}{\alpha}e^{-x} + c\,x^{-\alpha}e^{-x}$$

if $\alpha = 0$ $\quad e^x y = \ln x + c \quad y = \ln x\, e^{-x} + c e^{-x}$

c) $y' + 4xy = 2xy^2 + 2x$

$$y' = x\cdot(2y^2 + 2 - 4y) = 2x(y-1)^2$$

$$\frac{1}{(y-1)^2}\,dy = 2x\,dx$$

$$-\frac{1}{(y-1)} = x^2 + c$$

$$y = \frac{1}{-x^2-c} + 1$$

10E

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Solution(s):

From user: ar857

2005 I 10

$y'' - (a+b)y' + aby = e^{cx}$

a) $a \neq b \neq c \neq a$

$\lambda^2 - (a+b)\lambda + ab = 0 \Rightarrow \lambda_1 = a \quad \lambda_2 = b$

$y_c = c_1 e^{ax} + c_2 e^{bx}$

$y_p = K e^{cx} \quad Ly_p = Ke^{cx} \cdot (c^2 - (a+b)c + ab) = e^{cx}$

$\Rightarrow K = (c^2-(a+b)c+ab)^{-1} = (c^2 - ac - bc + ab) = [(c-a)(c-b)]^{-1}$

$y(0) = c_1 + c_2 + K = 0 \Leftrightarrow c_1(c-a)(c-b) + c_2(c-a)(c-b) + 1 = 0$

$\Downarrow$

$c_1 = \frac{-1}{(c-a)(c-b)} - c_2$

$y'(0) = ac_1 + bc_2 + \frac{c}{(c-a)(c-b)} = 0$

$\frac{-a}{(c-a)(c-b)} + (b-a)c_2 + \frac{c}{(c-a)(c-b)} = \frac{1}{(c-b)} + (b-a)c_2 = 0 \Rightarrow c_2 = \frac{1}{(a-b)(c-b)}$

$c_1 = \frac{1}{(c-a)(b-a)}$

$(b-c)+(c-a)+(a-b) = 0$ ✓

$y = \frac{1}{(c-a)(b-a)} e^{a} + \frac{1}{(c-b)(a-b)} e^{b} + \frac{1}{(c-a)(c-b)} e^{c}$

b) $a \neq b = c$

$y'' - (a+b)y' + aby = e^{bx} \qquad y_c = c_1 e^{ax} + c_2 e^{bx}$

$y_p = Kxe^{bx}$ ✓

$Ly_p = (2bke^{bx} + b^2xke^{bx}) + (Ke^{bx} + bxKe^{bx})(-a-b) + abKxe^{bx}$

$= (b-a)Ke^{bx} = e^{bx} \Rightarrow K = \frac{1}{b-a}$

$y = c_1 e^{ax} + c_2 e^{bx} + \frac{1}{b-a} x e^{bx}$

$y(0) = c_1 + c_2 + \cancel{} = 0 \Rightarrow c_1 = -c_2$

$y'(0) = ac_1 + bc_2 + \frac{1}{b-a} = 0 \Rightarrow (a-b)c_1 = \frac{1}{a-b} \quad c_1 = \frac{1}{(a-b)^2} \quad c_2 = \frac{-1}{(a-b)^2}$

$y = \frac{1}{(a-b)^2} e^{ax} - \frac{1}{(a-b)^2} e^{bx} + \frac{1}{(b-a)} x e^{bx}$

11F

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Solution(s):

From user: ar857

2005 I 11

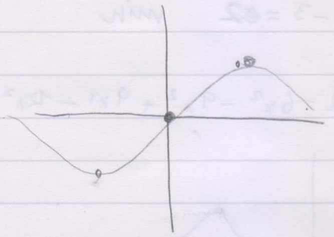
a) $f=(x+x^3)e^{-x^2}$

$f' = (1+3x^2-2x^2-2x^4)\,e^{-x^2} = -(2x^4-x^2-1)\,e^{-x^2} = -e^{-x^2}\cdot(x^2-1)(2x^2+1)$

$\overset{\text{max}}{x=1}$ $\overset{\text{min}}{x=-1}$

$f'' = -e^{-x^2}(8x^3 \cancel{-2x} - 4x^5 + 2x^3 \cancel{-2x}) = -2e^{-x^2}\cdot x^3(5-2x^2)$

b)



$x^2 \cdot xe^{-x^2}$

c) i) $=0$

ii) $\int_0^\infty xe^{-x^2} + x^3e^{-x^2} = -\frac{1}{2}e^{-x^2} + x^2\cdot -\frac{1}{2}e^{-x^2} - \frac{1}{2}e^{-x^2} = -(\frac{x^2}{2}+1)e^{-x^2}\Big]_0^\infty = 1$

iii) $\int_0^\infty f(x)\frac{df(x)}{dx}\,dx = f(x)\cdot f(x) - \int_0^\infty f(x)\frac{df}{dx}$

$= \frac{1}{2}\left((x+x^3)e^{-x^2}\cdot(x+x^3)e^{-x^2}\right)\Big|_0^\infty = \frac{1}{2}\cdot(0-0) = 0$

12F

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