2004 Mathematics (1)

1A

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2A*

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3B*

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4B

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8D

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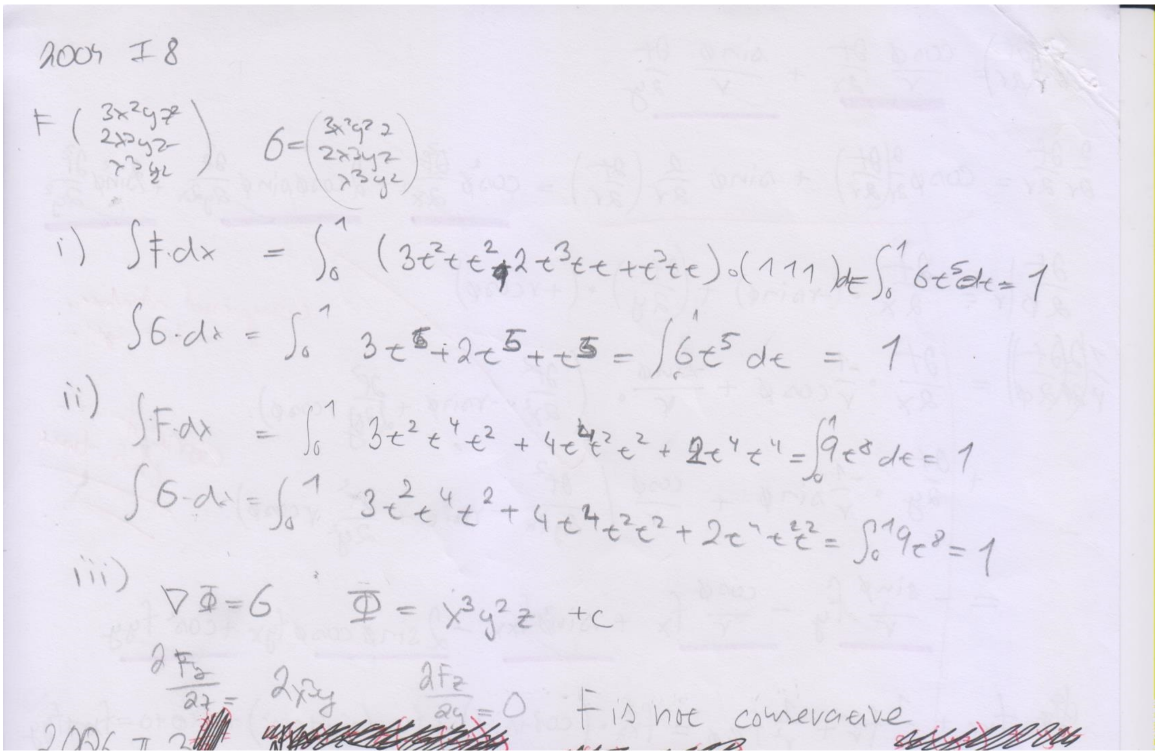
Solution(s):

From user: ar857



2004 I 8

$F \begin{pmatrix} 3x^2yz \\ 2x^3yz \\ x^3y^2 \end{pmatrix}$ $G = \begin{pmatrix} 3x^2y^2z \\ 2x^3yz \\ x^3y^2 \end{pmatrix}$

i) $\int F \cdot dx = \int_0^1 (3t^2 t t^2 + 2t^3 t t + t^3 t t) \cdot (1\,1\,1)\, dt = \int_0^1 6t^5 dt = 1$

$\int G \cdot dx = \int_0^1 3t^5 + 2t^5 + t^5 = \int_0^1 6t^5\, dt = 1$

ii) $\int F \cdot dx = \int_0^1 3t^2 t^4 t^2 + 4t^4 t^2 t^2 + 2t^4 t^4 = \int_0^1 9t^8\, dt = 1$

$\int G \cdot dx = \int_0^1 3t^2 t^4 t^2 + 4t^4 t^2 t^2 + 2t^4 t^2 t^2 = \int_0^1 9t^8 = 1$

iii) $\nabla \Phi = G$ $\Phi = x^3 y^2 z + c$

$\frac{\partial F_z}{\partial z} = 2x^3y$ $\frac{\partial F_z}{\partial y} = 0$ F is not conservative

99E

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10E

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11F

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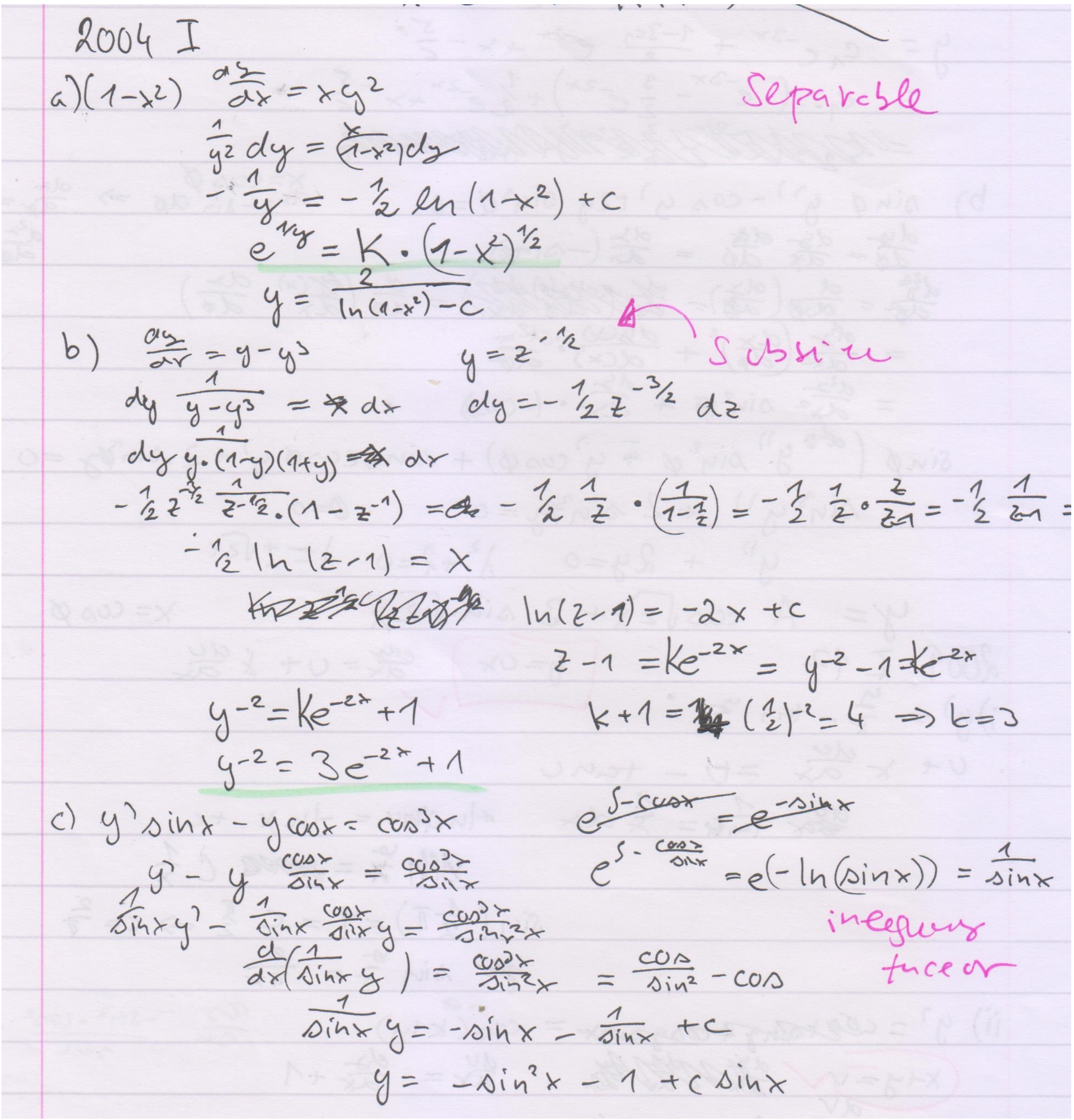

2004 I

a) $(1-x^2)\frac{dy}{dx} = xy^2$ Separable

$\frac{1}{y^2}dy = \frac{x}{(1-x^2)}dx$

$-\frac{1}{y} = -\frac{1}{2}\ln(1-x^2)+c$

$e^{1/y} = K\cdot(1-x^2)^{1/2}$

$y = \frac{2}{\ln(1-x^2)-c}$

b) $\frac{dy}{dx} = y - y^3$ $y = z^{-1/2}$ Subst.

$dy\frac{1}{y-y^3} = dx$ $dy = -\frac{1}{2}z^{-3/2}dz$

$dy\frac{1}{y\cdot(1-y)(1+y)} = dx$

$-\frac{1}{2}z^{-3/2}\frac{1}{z^{-1/2}\cdot(1-z^{-1})} = -\frac{1}{2}\frac{1}{z}\cdot\left(\frac{1}{1-\frac{1}{z}}\right) = -\frac{1}{2}\frac{1}{z}\cdot\frac{z}{z-1} = -\frac{1}{2}\frac{1}{z-1} =$

$-\frac{1}{2}\ln(z-1) = x$

$\ln(z-1) = -2x+c$

$z-1 = Ke^{-2x} = y^{-2}-1 = Ke^{-2x}$

$y^{-2} = Ke^{-2x}+1$ $K+1 = (\frac{1}{2})^{-2} = 4 \Rightarrow K=3$

$y^{-2} = 3e^{-2x}+1$

c) $y'\sin x - y\cos x = \cos^3 x$ $e^{\int -\frac{\cos x}{\sin x}} = e(-\ln(\sin x)) = \frac{1}{\sin x}$

$y' - y\frac{\cos x}{\sin x} = \frac{\cos^3 x}{\sin x}$

$\frac{1}{\sin x}y' - \frac{1}{\sin x}\frac{\cos x}{\sin x}y = \frac{\cos^3 x}{\sin^2 x}$ integrating factor

$\frac{d}{dx}\left(\frac{1}{\sin x}y\right) = \frac{\cos^3 x}{\sin^2 x} = \frac{\cos}{\sin^2} - \cos$

$\frac{1}{\sin x}y = -\sin x - \frac{1}{\sin x} + c$

$y = -\sin^2 x - 1 + c\sin x$

12F

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Solution(s):

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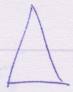

a) $\int_C \rho \, dV = \int_0^{2\pi}\int_0^h\int_0^{a(1-\frac{z}{h})} \rho_0 \left(1+\frac{r}{a}\right) r \, dr \, dz \, d\phi$

$= 2\pi\rho_0 \int_0^h \left[\frac{1}{2}r^2 + \frac{1}{3a}r^3\right]_0^{a(1-\frac{z}{h})} dz$

$= 2\pi\rho_0 \int_0^h \frac{a^2}{2}\cdot\left(1+\frac{z^2}{h^2}-2\frac{z}{h}\right) + \frac{a^2}{3}\cdot\left(1+3\frac{z^2}{h^2}-3\frac{z}{h}-\frac{z^3}{h^3}\right)$

$= 2\pi\rho_0 \int_0^h a^2 \left(\frac{5}{6}+\frac{3}{2}\frac{z^2}{h^2}-2\frac{z}{h}-\frac{z^3}{h^3 \, 3}\right) dz$

$= 2\pi\rho_0 a^2\cdot\left(\frac{5}{6}h+\frac{1}{2}h-h-\frac{1}{12}h\right) = \frac{2\pi\rho_0 a^2 h}{4} = \frac{\pi\rho_0 a^2 h}{2}$

b) $d = \frac{2}{\pi\rho_0 a^2 h}\cdot 2\pi\rho_0 \int_0^h\int_0^{a(1-\frac{z}{h})} zr + \frac{zr^2}{a} \, dr \, dz$

$d = \frac{4}{a^2 h} \left[\frac{5}{12}+\frac{3}{8}-\frac{2}{3}-\frac{1}{15}\right] a^2 h^2$

$d = h\cdot\left[\frac{5}{3}+\frac{3}{2}-\frac{8}{3}-\frac{4}{15}\right] = \frac{50+45-80-8}{30}h = \frac{7}{30}h$