

2004 Mathematics (1)

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1A

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2A*

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3B*

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6C

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7D*

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8D

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Solution(s):

From user: ar857

1005 I 8

$F = \begin{pmatrix} 3x^2yz^2 \\ 2xy^2z \\ x^3yz \end{pmatrix} \quad \phi = \begin{pmatrix} 3x^2yz^2 \\ 2x^2yz \\ x^3yz \end{pmatrix}$

i) $\int F \cdot dx = \int_0^1 (3t^2 + t^2 + 2t^3t + t^2t) \cdot (1, 1, 1) dt = \int_0^1 6t^5 dt = 1$
 $\int \phi \cdot dx = \int_0^1 3t^5 + 2t^5 + t^5 = \int_0^1 6t^5 dt = 1$

ii) $\int F \cdot dx = \int_0^1 3t^2t^4t^2 + 4t^4t^2t^2 + 2t^4t^4 = \int_0^1 9t^8 dt = 1$
 $\int \phi \cdot dx = \int_0^1 3t^2t^4t^2 + 4t^4t^2t^2 + 2t^4t^4 = \int_0^1 9t^8 dt = 1$

iii) $\nabla \phi = F \quad \phi = x^3y^2z + c$

$\frac{\partial F_x}{\partial z} = 2xy \quad \frac{\partial F_z}{\partial y} = 0 \quad F \text{ is not conservative}$

99E

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10E

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11F

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Solution(s):

From user: ar857

2004 I

a) $(1-x^2) \frac{dy}{dx} = xy^2$

Separable

$$\frac{1}{y^2} dy = \frac{x}{1-x^2} dx$$

$$-\frac{1}{y} = -\frac{1}{2} \ln(1-x^2) + C$$

$$e^{1/y} = K \cdot (1-x^2)^{1/2}$$

$$y = \frac{2}{\ln(1-x^2) - C}$$

b) $\frac{dy}{dx} = y - y^3$

$$y = z^{-1/2}$$

Substitution

$$dy \frac{1}{y-y^3} = dx \quad dy = -\frac{1}{2} z^{-3/2} dz$$

$$dy \frac{1}{y(1-y^2)(1+y)} = dx$$

$$-\frac{1}{2} z^{-1/2} \frac{1}{z^{-1/2} \cdot (1-z^{-1})} = -\frac{1}{2} \frac{1}{z} \cdot \left(\frac{1}{1-\frac{1}{z}} \right) = -\frac{1}{2} \frac{1}{z} \cdot \frac{z}{z-1} = -\frac{1}{2} \frac{1}{z-1} =$$

$$-\frac{1}{2} \ln(z-1) = x$$

$$\ln(z-1) = -2x + C$$

$$z-1 = Ke^{-2x} = y^{-2} - 1 = Ke^{-2x}$$

$$y^{-2} = Ke^{-2x} + 1$$

$$K+1 = \left(\frac{1}{2}\right)^{-2} = 4 \Rightarrow K=3$$

$$y^{-2} = 3e^{-2x} + 1$$

c) $y' \sin x - y \cos x = \cos^3 x$

$$e^{\int -\cos x} = e^{-\sin x}$$

$$e^{\int -\frac{\cos x}{\sin x}} = e^{(-\ln(\sin x))} = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} y' - \frac{1}{\sin x} \frac{\cos x}{\sin x} y = \frac{\cos^3 x}{\sin^2 x}$$

$$\frac{d}{dx} \left(\frac{1}{\sin x} y \right) = \frac{\cos^3 x}{\sin^2 x} = \frac{\cos}{\sin^2} - \cos$$

integrating factor

$$\frac{1}{\sin x} y = -\sin x - \frac{1}{\sin x} + C$$

$$y = -\sin^2 x - 1 + C \sin x$$

12F

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Solution(s):

From user: ar857

2004 I 2



$$\begin{aligned}
 a) \int_C \rho \, dV &= \int_0^{2\pi} \int_0^h \int_0^{a(1-\frac{z}{h})} \rho_0 \left(1 + \frac{r}{a}\right) r \, dr \, dz \, d\phi \\
 &= 2\pi \rho_0 \int_0^h \left[\frac{1}{2} r^2 + \frac{1}{3a} r^3 \right]_0^{a(1-\frac{z}{h})} dz \\
 &= 2\pi \rho_0 \int_0^h \frac{a^2}{2} \cdot \left(1 + \frac{z^2}{h^2} - 2\frac{z}{h}\right) + \frac{a^2}{3} \cdot \left(1 + \frac{z^2}{h^2} - \frac{z^2}{h^2} - \frac{z^3}{h^3}\right) dz \\
 &= 2\pi \rho_0 \int_0^h \frac{a^2}{2} \left(\frac{5}{6} + \frac{3}{2} \frac{z^2}{h^2} - 2\frac{z}{h} - \frac{z^3}{h^3} \right) dz \\
 &= 2\pi \rho_0 a^2 \cdot \left(\frac{5}{6} h + \frac{3}{2} h - h - \frac{1}{12} h \right) = 2\pi \rho_0 a^2 h \cdot \frac{4}{4} = \frac{\pi \rho_0 a^2 h}{2} \\
 b) d &= \frac{2}{\pi \rho_0 a^2 h} \cdot 2\pi \rho_0 \int_0^h \int_0^{a(1-\frac{z}{h})} z r + \frac{z r^2}{a} \, dr \, dz \\
 d &= \frac{4}{a^2 h} \int_0^h \left[\frac{5}{12} + \frac{3}{8} - \frac{z}{3} - \frac{1}{15} \right] a^2 h^2 dz \\
 d &= \frac{4}{a^2 h} h \cdot \left[\frac{5}{3} + \frac{3}{2} - \frac{8}{3} - \frac{4}{15} \right] = \frac{50 + 45 - 80 - 8}{30} h = \frac{7}{30} h
 \end{aligned}$$