# 2003 Mathematics (1)

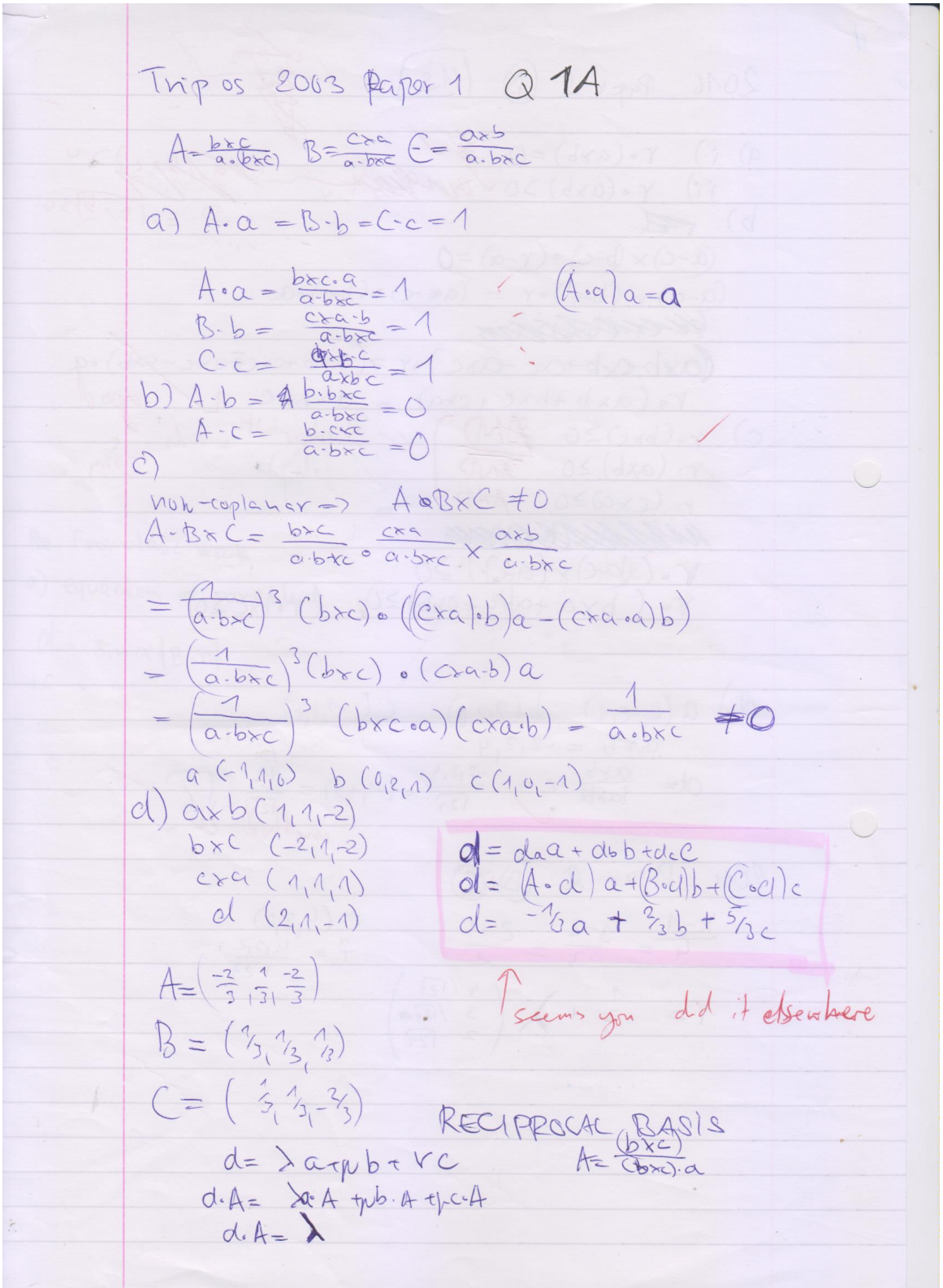
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## 1A

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### *Solution(s):*

From user: ar857




## 2A*

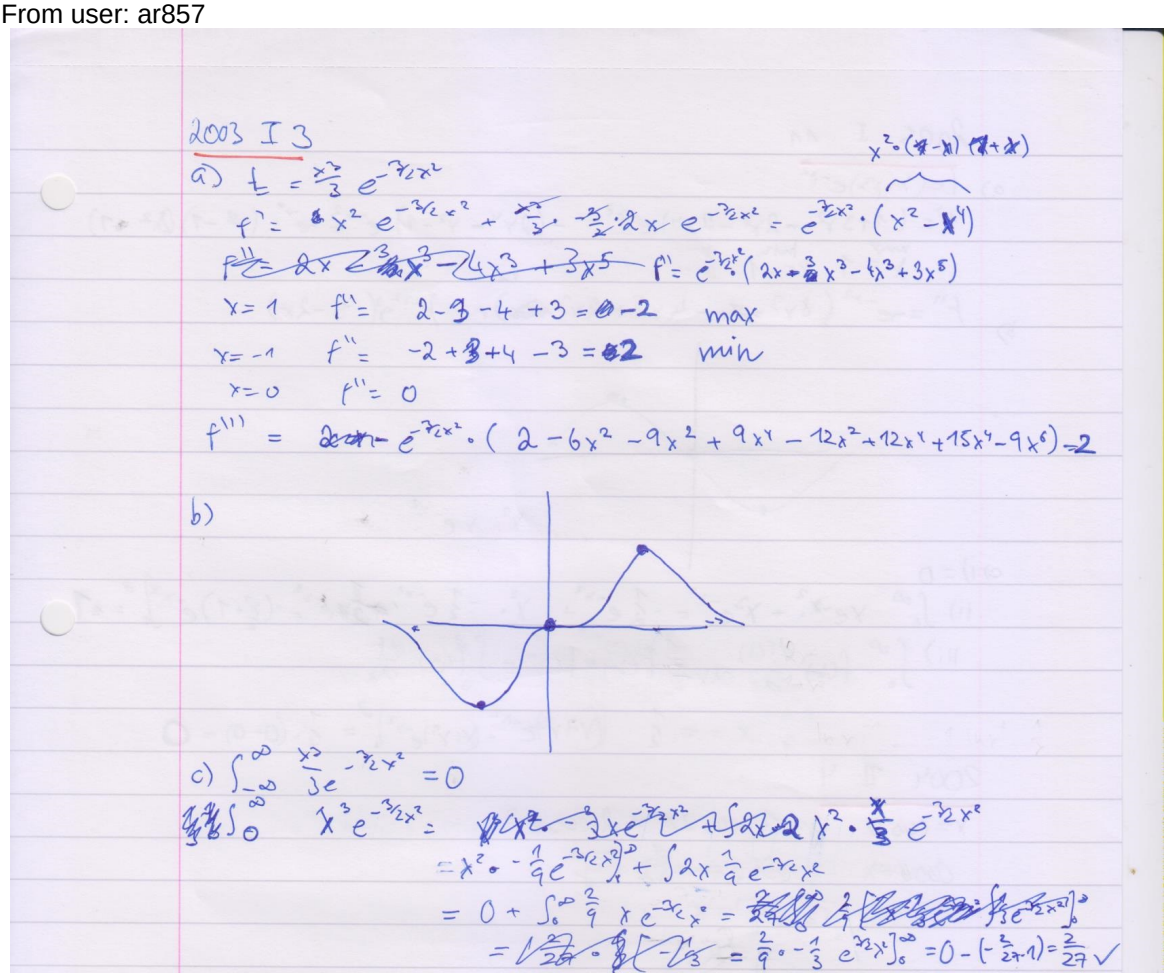
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## 3B

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### *Solution(s):*

From user: ar857

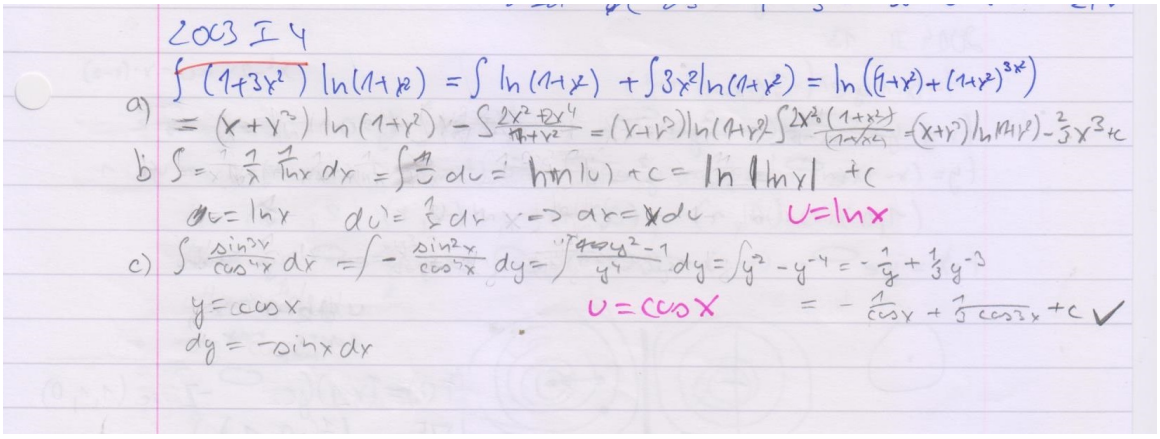



## 4B

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### *Solution(s):*

From user: ar857



## 5C

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## 6C*

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## 7D

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## 8D

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## 9E

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## 10E*

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## 11F

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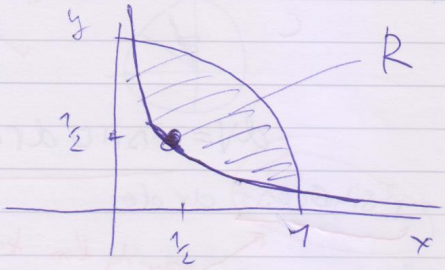
## 12F

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### *Solution(s):*

From user: ar857

2003 I 12 F



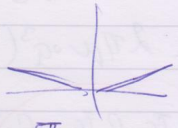

$r^2 = 1$ $\quad r^2 \cos\sigma \sin\sigma = \frac{1}{4}$

$\frac{1}{2} \sin 2\sigma = \frac{1}{4}$

$\sin 2\sigma = \frac{1}{2}$

$2\sigma = \frac{\pi}{6}$ or $2\pi = \frac{5\pi}{6}$

$\sigma = \frac{\pi}{12}$ or $\pi = \frac{5\pi}{12}$



intersection : $(r=1 \ \sigma = \frac{\pi}{12})$ and $(r=1 \ \sigma = \frac{5\pi}{12})$

$$\int_{\pi/12}^{\frac{5}{12}\pi} \int_{\frac{1}{4\cos\sigma\sin\sigma}}^{1} r^5 \cos\sigma \sin\sigma \, dr \, d\sigma$$

$$= \frac{1}{6} \int_{\pi/12}^{5/12\,\pi} \cos\sigma \sin\sigma \, d\sigma \quad \frac{1}{6} \int_{\pi/12}^{\frac{5\pi}{12}} \frac{1}{64 (\cos\sigma \sin\sigma)^2} \, d\sigma$$

$$= \frac{1}{12} \cdot \frac{1}{2} \cdot \left( \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) - \frac{1}{96} \int_{\pi/2}^{\frac{5\pi}{12}} \frac{1}{(\sin 2\sigma)^{-2}}$$

$$= \frac{\sqrt{3}}{24} + \frac{1}{96} \cdot \frac{1}{2} \left[ \cot\frac{5}{6}\pi - \cot\frac{1}{6}\pi \right]$$

$$= \frac{\sqrt{3}}{24} + \frac{1}{96} \cdot \frac{1}{2} \cdot -2\sqrt{3} = \sqrt{3}\left(\frac{1}{24} - \frac{1}{96}\right) = \frac{\sqrt{3}}{32}$$

$\frac{4}{96} - \frac{1}{96} = \frac{3}{96} = \frac{1}{32}$ ✓