2003 Mathematics (2)

1A

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

2A

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

3B

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

4B*

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

5C

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

6C

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

7D

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

8D*

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

9E

No image has yet been uploaded for this question

Solution(s):

From user: ar857

2003 II 9

a) $dU = TdS - pdV$

$H = U + pV$

$dH = TdS - pdV + pdV + Vdp = TdS + Vdp$

du is exact $\Rightarrow$ $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$ $\quad T = \left(\frac{\partial U}{\partial S}\right)_V$ $\quad -p = \left(\frac{\partial U}{\partial V}\right)_S$

dH is exact $\Rightarrow$ $\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p$ $\quad T = \left(\frac{\partial H}{\partial S}\right)_p$ $\quad V = \left(\frac{\partial H}{\partial p}\right)_S$

b) $c_p - c_v = \left(\frac{\partial H}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_V$ $\quad H = U + pV$

$= \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_V$

$U(T,p) \Rightarrow U(T,V)$

$\cancel{\left(\frac{\partial U}{\partial T}\right)_p = \frac{\partial U}{\partial S}\left(\frac{\partial S}{\partial T}\right) + \frac{\partial U}{\partial V}\frac{\partial V}{\partial T}}$

$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_V\left(\frac{\partial T}{\partial T}\right)_p + \left(\frac{\partial U}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p$

$c_p - c_v = \cancel{\left(\frac{\partial U}{\partial T}\right)_V} + \left(\frac{\partial U}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p - \cancel{\left(\frac{\partial U}{\partial T}\right)_V}$

$= \left(\frac{\partial V}{\partial T}\right)_p \cdot \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]$

for ideal gas

$c_p - c_v = p\left(\frac{\partial V}{\partial T}\right)_p = p \cdot \frac{Nk}{p} = Nk$ ✓

10E

No image has yet been uploaded for this question
No soution has yet been submitted for this question.

11F

No image has yet been uploaded for this question

Solution(s):

From user: ar857

2003 II 11

$y'' - 2y' + y = 2x\sin x$

$y_c = c_1 e^x + c_2 x e^x$

$y_p = (a+bx)\sin x + (c+dx)\cos x$

$y'_p = a\cos x + b\sin x + bx\cos x - c\sin x + d\cos x - dx\sin x$

$y''_p = -a\sin x + b\cos x + b\cos x - bx\sin x - c\cos x - d\sin x - d\sin x - dx\cos x$

$Ly_p = -a\sin x + b\cos x + b\cos x - bx\sin x - c\cos x - d\sin x - d\sin x - dx\cos x$
$\quad - 2a\cos x - 2b\sin x - 2bx\cos x + 2c\sin x - 2d\cos x + 2dx\sin x$ $+ a\sin x + bx\sin x + c\cos x + dx\cos x$

$$\Rightarrow \boxed{\begin{array}{ll} -2d - 2b + 2c = 0 & \Rightarrow c = 1 \\ 2b - 2a - 2d = 0 & \Rightarrow a = -1 \\ +2d = 2 & \Rightarrow d = 1 \\ -2b = 0 & \Rightarrow b = 0 \end{array}}$$

$y = c_1 e^x + c_2 x e^x - \sin x + (1+x)\cos x$

$y(0) = c_1 + 0 - 0 + 1 = 0 \quad \Rightarrow c_1 = -1$

$y'(0) = c_1 + c_2 - 1 + 1 = 0 \quad c_2 = 1$

$y = (x-1)e^{-x} - \sin x + (1+x)\cos x$

2003 II 11

12F*

No image has yet been uploaded for this question
No soution has yet been submitted for this question.