

2003 Mathematics (2)

This pdf was generated from questions and answers contributed by members of the public to Christopher Lester's tripos/example-sheet solution exchange site <http://cgl20.user.srcf.net/>. Nothing (other than raven authentication) prevents rubbish being uploaded, so this pdf comes with no warranty as to the correctness of the questions or answers contained. Visit the site, vote, and/or supply your own content if you don't like what you see here.

This pdf had url <http://cgl20.user.srcf.net/camcourse/paperpdf/31?withSolutions=1>.

This pdf was created on Sun, 03 Dec 2023 13:40:43 +0000.

1A

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

2A

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

3B

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

4B*

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

5C

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

6C

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

7D

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

8D*

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

9E

No image has yet been uploaded for this question

Solution(s):

From user: ar857

2003 II 9

$$a) \quad dU = Tds - pdv$$

$$H = U + pV$$

$$dH = Tds - pdv + pdv + Vdp = Tds + Vdp$$

$$dU \text{ is exact} \Rightarrow \left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v \quad T = \left(\frac{\partial U}{\partial s}\right)_v$$

$$-p = \left(\frac{\partial U}{\partial v}\right)_s$$

$$dH \text{ is exact} \Rightarrow \left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p \quad T = \left(\frac{\partial H}{\partial s}\right)_p$$

$$V = \left(\frac{\partial H}{\partial p}\right)_s$$

$$b) \quad c_p - c_v = \left(\frac{\partial H}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v \quad H = U + pV$$

$$= \left(\frac{\partial U}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v$$

$$U(T, p) \Rightarrow U(T, v)$$

$$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial s}\right)_p \frac{\partial s}{\partial T} + \left(\frac{\partial U}{\partial v}\right)_p \frac{\partial v}{\partial T}$$

$$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p + \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_v + \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

$$c_p - c_v = \left(\frac{\partial U}{\partial T}\right)_v + \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v$$

$$= \left(\frac{\partial v}{\partial T}\right)_p \cdot \left[\left(\frac{\partial U}{\partial v}\right)_T + p \right]$$

for ideal gas

$$c_p - c_v = p \left(\frac{\partial v}{\partial T}\right)_p = p \cdot \frac{Nk}{p} = Nk \quad \checkmark$$

10E

No image has yet been uploaded for this question
No solution has yet been submitted for this question.

11F

No image has yet been uploaded for this question

Solution(s):

From user: ar857

ROOS II 11

$$y'' - 2y' + y = 2x \sin x$$

$$y_c = c_1 e^x + c_2 x e^x$$

$$y_p = (a+bx) \sin x + (c+dx) \cos x$$

$$y'_p = a \cos x + b \sin x + b x \cos x - c \sin x + d \cos x - d x \sin x$$

$$y''_p = -a \sin x + b \cos x + b \cos x - b x \sin x - c \cos x - d \sin x - d \sin x - d x \cos x$$

$$L y_p = -a \sin x + b \cos x + b \cos x - b x \sin x - c \cos x - d \sin x - d \sin x - d x \cos x - 2a \cos x - 2b \sin x - 2b x \cos x + 2c \sin x - 2d \cos x + 2d x \sin x + a \sin x + b x \sin x + c \cos x + d x \cos x$$

$$\Rightarrow \begin{cases} -2d = -2b + 2c = 0 & \Rightarrow c = 1 \\ 2b - 2a - 2d = 0 & \Rightarrow a = -1 \\ 2d = 2 & \Rightarrow d = 1 \\ -2b = 0 & \Rightarrow b = 0 \end{cases}$$

$$y = c_1 e^x + c_2 x e^x + \sin x + (1+x) \cos x$$

$$y(0) = c_1 + 0 - 0 + 1 = 0 \Rightarrow c_1 = -1$$

$$y'(0) = c_1 + c_2 - 1 + 1 = 0 \Rightarrow c_2 = 1$$

$$y = (x-1) e^{-x} + \sin x + (1+x) \cos x$$

ROOS II 11

12F*

No image has yet been uploaded for this question
No solution has yet been submitted for this question.