

2003 Mathematics (2)

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1A

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2A

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3B

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4B*

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5C

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7D

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8D*

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9E

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Solution(s):

From user: ar857

2003 II 9

a) $dU = Tds - pdv$

$H = U + pV$

$dH = Tds - pdv + pdv + Vdp = Tds + Vdp$

du is exact $\Rightarrow \left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$ $T = \left(\frac{\partial U}{\partial s}\right)_v$
 $-p = \left(\frac{\partial U}{\partial v}\right)_s$

dH is exact $\Rightarrow \left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial V}{\partial s}\right)_p$ $T = \left(\frac{\partial H}{\partial s}\right)_p$
 $V = \left(\frac{\partial H}{\partial p}\right)_s$

b) $c_p - c_v = \left(\frac{\partial H}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v$ $H = U + pV$

$= \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial v}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v$

$U(T, p) \Rightarrow U(T, v)$

~~$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial s}\right)_p \left(\frac{\partial s}{\partial T}\right)_p + \left(\frac{\partial U}{\partial v}\right)_p \left(\frac{\partial v}{\partial T}\right)_p$~~

$\left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p + \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_v + \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$

$c_p - c_v = \left(\frac{\partial U}{\partial T}\right)_v + \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p + p\left(\frac{\partial v}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v$
 $= \left(\frac{\partial v}{\partial T}\right)_p \left[\left(\frac{\partial U}{\partial v}\right)_T + p \right]$

for ideal gas

$c_p - c_v = p\left(\frac{\partial v}{\partial T}\right)_p = p \cdot \frac{Nk}{p} = Nk$ ✓

10E

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11F

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Solution(s):

From user: ar857

ROOS II 11

$$y'' - 2y' + y = 2x \sin x$$

$$y_c = c_1 e^x + c_2 x e^x$$

$$y_p = (a+bx) \sin x + (c+dx) \cos x$$

$$y'_p = a \cos x + b \sin x + b x \cos x - c \sin x + d \cos x - d x \sin x$$

$$y''_p = -a \sin x + b \cos x + b \cos x - b x \sin x - c \cos x - d \sin x - d \sin x - d x \cos x$$

$$L y_p = -a \sin x + b \cos x + b \cos x - b x \sin x - c \cos x - d \sin x - d \sin x - d x \cos x - 2a \cos x - 2b \sin x - 2b x \cos x + 2c \sin x - 2d \cos x + 2d x \sin x + a \sin x + b x \sin x + c \cos x + d x \cos x$$

$$\Rightarrow \begin{cases} -2b + 2c = 0 & \Rightarrow c = b \\ 2b - 2a - 2d = 0 & \Rightarrow a = -1 \\ 2a + 2d = 2 & \Rightarrow d = 1 \\ -2b = 0 & \Rightarrow b = 0 \end{cases}$$

$$y = c_1 e^x + c_2 x e^x + \sin x + (1+x) \cos x$$

$$y(0) = c_1 + 0 - 0 + 1 = 0 \Rightarrow c_1 = -1$$

$$y'(0) = c_1 + c_2 - 1 + 1 = 0 \Rightarrow c_2 = 1$$

$$y = (x-1) e^{-x} - \sin x + (1+x) \cos x$$

ROOS II 11

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