

2002 Mathematics (2)

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1A

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2A*

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3B

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Solution(s):

From user: ar857

2002 II 3

$$x = r \sin \sigma \cos \phi$$

$$y = r \sin \sigma \sin \phi$$

$$z = r \cos \sigma$$

a) $\int_0^{\pi} \int_0^{\pi} \int_0^a 2r \sin \sigma + r^2 \sin \sigma \cos \sigma \, dr \, d\sigma \, d\phi$

$$= 2\pi \mu \int_0^{\pi} \int_0^{\pi} \left[\frac{2}{3} a^3 \sin \sigma + \frac{1}{3} a^3 \sin \sigma \cos \sigma \right] d\sigma$$

arguably this term, odd in σ , range even in σ , was never going to contribute.

$$= 2\pi \mu a^3 \left(\frac{4}{3} + 0 \right) = \frac{8}{3} \pi \mu a^3$$

b) $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^a r^3 \sin \sigma \cos \phi \sin \phi \, dr \, d\sigma \, d\phi$

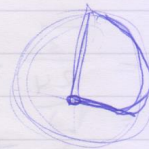
$$= \frac{\lambda a^5}{5} \int_0^{\pi/2} \int_0^{\pi/2} \sin^3 \sigma \cos \phi \sin \phi \, d\sigma \, d\phi$$

$$= \frac{\lambda a^5}{5} \int_0^{\pi/2} \cos \phi \sin \phi \left(\sin \sigma - \sin \sigma \cos^2 \sigma \right) d\phi$$

$$= \frac{\lambda a^5}{5} \int_0^{\pi/2} \cos \phi \sin \phi \left[(0 - (-1)) + (0 - \frac{1}{3}) \right] d\phi$$

$$= \frac{2\lambda a^5}{15} \int_0^{\pi/2} \cos \phi \sin \phi \, d\phi = \frac{2\lambda a^5}{15} \cdot \frac{1}{2} = \frac{a^5 \lambda}{15}$$

$$2 + \frac{r \cos \phi}{r}$$



$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^a$$

$$\lambda r \cos \phi r \sin^4 \phi r^2 \sin \phi$$

4B

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5C

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6C*

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7D

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Solution(s):

From user: ar857

2002 II 7

a) $y'' + 5y' + 6y = 6x$
 $y_h = c_1 e^{-3x} + c_2 e^{-2x}$
 $y_p = kx + B$
 $L y_p = 0 + 5k + 6kx + 6B = 6x \Rightarrow k = 1 \quad \begin{matrix} 5+6B=0 \\ B = -5/6 \end{matrix}$
 $y = c_1 e^{-3x} + c_2 e^{-2x} + x - \frac{5}{6}$
 $y'(0) = -3c_1 - 2c_2 + 1 = 0 \Rightarrow c_2 = \frac{-3c_1 + 1}{2}$
 $y = c_1 \cdot (e^{-3x} - \frac{3}{2} e^{-2x}) + \frac{1}{2} e^{-2x} + x - \frac{5}{6}$

b) $\sin \theta y'' - \cos \theta y' + \lambda y \sin^3 \theta = 0 \quad x = \cos \theta \quad \frac{dy}{d\theta} = -\sin \theta \quad \frac{d^2 y}{d\theta^2} = -\cos \theta$
 $\sin \theta \frac{d}{d\theta} \left(\frac{dy}{d\theta} \frac{dx}{d\theta} \right) - \cos \theta \left(\frac{dx}{d\theta} \frac{dy}{d\theta} \right) + \lambda y \sin^3 \theta = 0$
 $\sin \theta \left(\frac{dx}{d\theta} \frac{d^2 y}{d\theta^2} + \frac{d^2 x}{d\theta^2} \frac{dy}{d\theta} \right) - \cos \theta \frac{dx}{d\theta} \frac{dy}{d\theta} + \lambda y \sin^3 \theta = 0$
 $\sin \theta \frac{d^2 y}{d\theta^2} - \cos \theta \frac{dx}{d\theta} \frac{dy}{d\theta} + \cos \theta \sin \theta \frac{dx}{d\theta} \frac{dy}{d\theta} + \lambda y \sin^3 \theta = 0$
 $\sin \theta \frac{d^2 y}{d\theta^2} + \lambda y \sin^3 \theta = 0 \quad \lambda^2 + 2 = 0 \Rightarrow \lambda = \pm \sqrt{2} i$
 $y = c_1 \cos \sqrt{2} x + c_2 \sin \sqrt{2} x \text{ where } x = \cos \theta$

8D

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9E

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10E*

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11F

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