# 2002 Mathematics (2)



## 1A

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## 2A*

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## 3B

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### *Solution(s):*

From user: ar857

2002 II 3

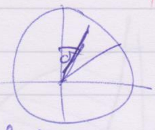
$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$\mu\int\left(2+\frac{z}{r}\right)dV$ ✓

C



$dV = r^2\sin\theta\, dr\, d\theta\, d\phi$

a)

$2\pi\mu\int_0^{\pi}\int_0^{a} 2r^2\sin\theta + r^2\sin\theta\cos\theta \, dr\, d\theta$

$= 2\pi\mu\int_0^{\pi} \frac{2}{3}a^3\sin\theta + \frac{1}{3}a^3\sin\theta\cos\theta$

$= 2\pi\mu\cdot a^3\left(\frac{4}{3}+0\right) = \frac{8}{3}\pi\mu a^3$

arguably this term, odd in z, range even in z, was never going to contribute.

b)

$\lambda\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{a} r^2\sin\theta\; r^2\cos\phi\sin\phi \, dr\, d\theta\, d\phi$ ③ ✓

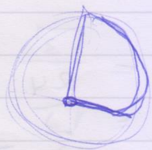
$= \frac{\lambda a^5}{5}\int_0^{\pi/2}\int_0^{\pi/2}\sin^3\theta\cos\phi\sin\phi$

$= \frac{\lambda a^5}{5}\int_0^{\pi/2}\int_0^{\pi/2}\cos\phi\sin\phi\left(\sin\theta - \sin\theta\cos^2\theta\right)$

$= \frac{\lambda a^5}{5}\int_0^{\pi/2}\cos\phi\sin\phi\left[(0--1)+\left(\frac{0}{3}-\frac{1}{3}\right)\right]$

$= \frac{2\lambda a^5}{15}\int_0^{\pi/2}\cos\theta\sin\theta = \frac{2\lambda a^5}{15}\cdot\frac{1}{2} = \frac{a^5\lambda}{15}$

$2 + \frac{r\cos\phi}{r}$



$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{a}$ ✓

$\lambda\; r\cos\phi\, r\sin\phi\; r^2\sin\phi$

# 4B

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# 5C

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## 6C*

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## 7D

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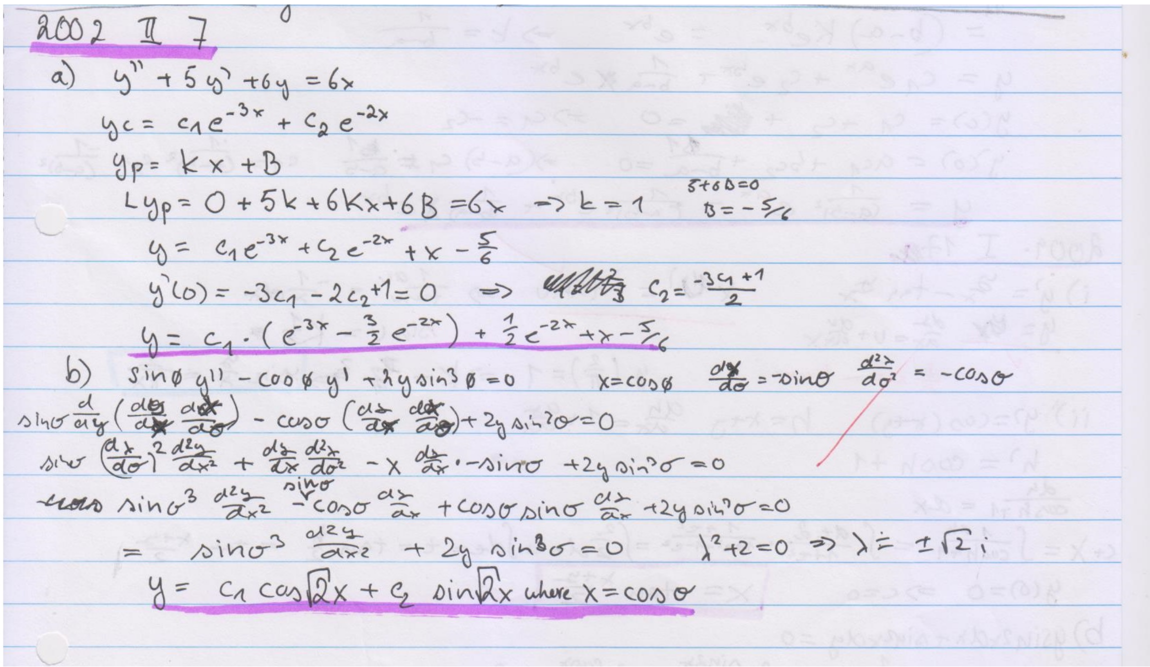

2002 II 7

a) $y'' + 5y' + 6y = 6x$

$y_c = c_1 e^{-3x} + c_2 e^{-2x}$

$y_p = kx + B$

$Ly_p = 0 + 5k + 6kx + 6B = 6x \Rightarrow k = 1$ $\quad 5+6B=0,\ B = -5/6$

$y = c_1 e^{-3x} + c_2 e^{-2x} + x - \frac{5}{6}$

$y'(0) = -3c_1 - 2c_2 + 1 = 0 \Rightarrow c_2 = \frac{-3c_1+1}{2}$

$y = c_1 \cdot (e^{-3x} - \frac{3}{2} e^{-2x}) + \frac{1}{2} e^{-2x} + x - 5/6$

b) $\sin\phi\, y'' - \cos\phi\, y' + 2y \sin^3\phi = 0$ $\quad x = \cos\phi$ $\quad \frac{dx}{d\theta} = -\sin\theta$ $\quad \frac{d^2x}{d\theta^2} = -\cos\theta$

$\sin\theta \frac{d}{dy}\left(\frac{dy}{dx}\frac{dx}{d\theta}\right) - \cos\theta \left(\frac{dy}{dx}\frac{dx}{d\theta}\right) + 2y\sin^3\theta = 0$

$\sin\theta \left(\frac{dx}{d\theta}\right)^2 \frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{d^2x}{d\theta^2} - x\frac{dy}{dx}\cdot -\sin\theta + 2y\sin^3\theta = 0$

$\sin\theta^3 \frac{d^2y}{dx^2} - \cos\theta \frac{dy}{dx}\sin\theta + \cos\theta\sin\theta \frac{dy}{dx} + 2y\sin^3\theta = 0$

$= \sin\theta^3 \frac{d^2y}{dx^2} + 2y\sin^3\theta = 0$ $\quad \lambda^2 + 2 = 0 \Rightarrow \lambda = \pm\sqrt{2}i$

$y = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x$ where $x = \cos\theta$

## 8D

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## 9E

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## 10E*

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## 11F

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## 12F

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