2001 Mathematics (2)

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1A

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2A*

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3B

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4B

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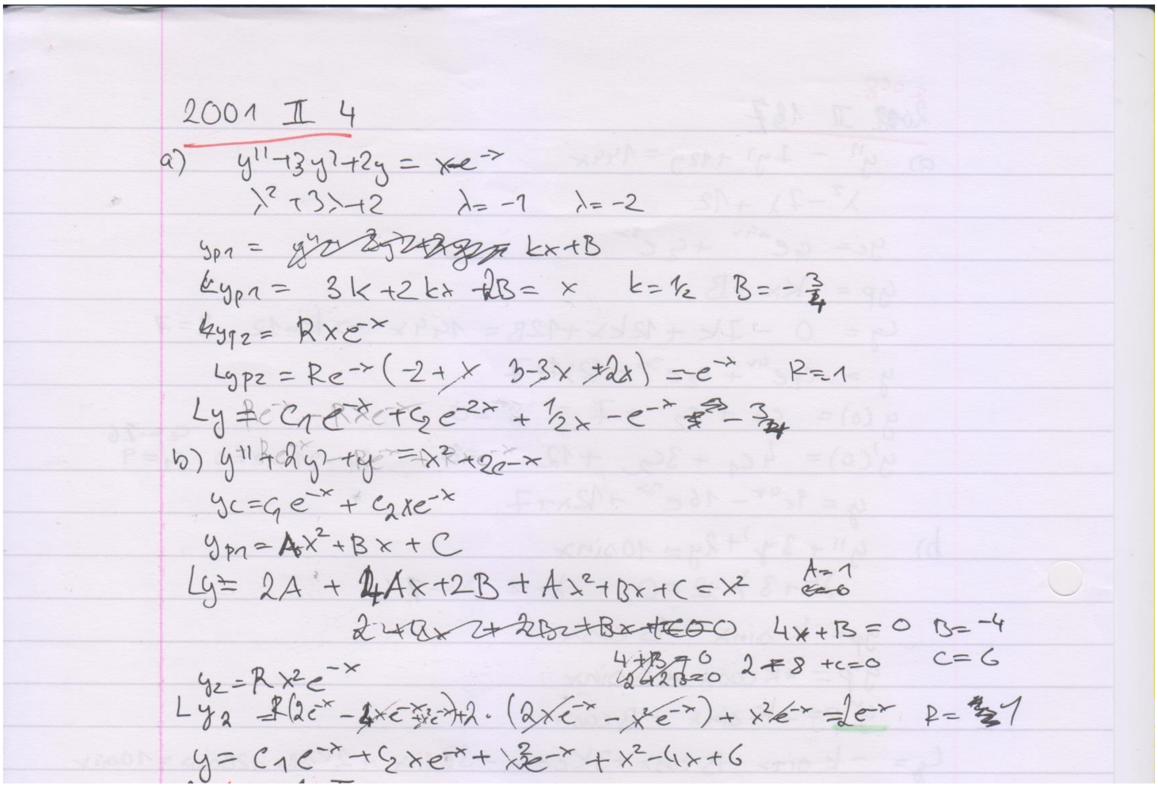
Solution(s):

From user: ar857



2001 II 4

a) $y'' + 3y' + 2y = xe^{-x}$

$\lambda^2 + 3\lambda + 2 \quad \lambda = -1 \quad \lambda = -2$

$y_{p1} = kx + B$

$Ly_{p1} = 3k + 2kx + 2B = x \quad k = 1/2 \quad B = -\frac{3}{4}$

$y_{p2} = Rxe^{-x}$

$Ly_{p2} = Re^{-x}(-2 + x \;\; 3 - 3x + 2x) = -e^{-x} \quad R = 1$

$y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{2}x - e^{-x} - \frac{3}{4}$

b) $y'' + 2y' + y = x^2 + 2e^{-x}$

$y_c = c_1 e^{-x} + c_2 x e^{-x}$

$y_{p1} = Ax^2 + Bx + C$

$Ly = 2A + 4Ax + 2B + Ax^2 + Bx + C = x^2 \quad A = 1$

$4x + Bx = 0 \quad B = -4$

$4 + B = 0 \quad 2 - 8 + c = 0 \quad C = 6$

$y_2 = Rx^2 e^{-x}$

$Ly_2 = R(2e^{-x} - 4xe^{-x} + x^2e^{-x}) + 2\cdot(2xe^{-x} - x^2 e^{-x}) + x^2 e^{-x} = 2e^{-x} \quad R = 1$

$y = c_1 e^{-x} + c_2 x e^{-x} + x^2 e^{-x} + x^2 - 4x + 6$

5C

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6C*

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7D

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8D

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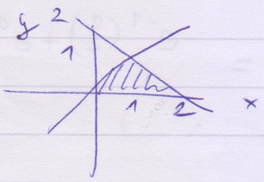
Solution(s):

From user: ar857

2001 II 8D



$$\int_0^1 \int_y^{2-y} (2x^2+y)\,dx\,dy$$

$$= \int_0^1 \frac{2}{3}(8-y^3+6y^2-12y-y^3)+2y-2y^2\,dy$$

$$= \frac{16}{3} - \frac{2}{12} + \frac{12}{9} - \frac{12}{3} - \frac{2}{12} + 1 - \frac{2}{3} = \frac{16+4-12}{3} = \frac{8}{3}$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{1+\cos\theta} r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= 2\pi \cdot \int_0^{\pi} \frac{1}{3}(1+\cos^3+3\cos^2+3\cos)\sin\theta$$

$$= 2\pi \cdot \left(-\frac{1}{3}\cos\pi + [illegible] - \frac{1}{3}\cos^3\pi - [illegible]\right)\cdot -2 = \frac{8}{3}\pi$$

9E

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Solution(s):

From user: ar857

$z^2 = 2x^2 + y^2$ $\quad z^2 = 2x^2 + x^2 + u^2 = 3x^2 + u^2 \Rightarrow 3x^2 = z^2 + u^2$, $\quad x = \sqrt{\frac{z^2+u^2}{3}}$

a) $u^2 = x^2 - y^2$ $\quad z^2 - 2u^2 = 3y^2$

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i) $\left(\frac{\partial x}{\partial z}\right)_u = \frac{1}{2}\frac{1}{x}\cdot\frac{2}{3}z$

$\left(\frac{\partial z}{\partial x}\right)_y = \frac{1}{2}\frac{1}{z}\cdot 4x$ $\Big\}$ $\left(\frac{\partial x}{\partial z}\right)_u\left(\frac{\partial z}{\partial x}\right)_y = \frac{2}{3}$

$\left(\frac{\partial y}{\partial z}\right)_u = \frac{1}{2}\frac{1}{y}\cdot\frac{2}{3}z$

$\left(\frac{\partial z}{\partial y}\right)_x = \frac{1}{2}\frac{1}{z}\cdot 2y$ $\Big\}$ $\left(\frac{\partial y}{\partial z}\right)_u\left(\frac{\partial z}{\partial y}\right)_x = \frac{1}{3}$

$\frac{2}{3} + \frac{1}{3} = 1$ ✓

ii) $\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{2}\frac{1}{x}\cdot\frac{-2y}{2}$

$\left(\frac{\partial x}{\partial u}\right)_z = \frac{1}{2}\frac{1}{x}\cdot\frac{2}{3}u$

$\left(\frac{\partial u}{\partial z}\right)_y = \frac{1}{2}\frac{1}{u}\cdot\frac{2}{2}z$

$\left(\frac{\partial u}{\partial x}\right)_y = \frac{1}{2}\frac{1}{u}\cdot 2x$

$\left(\frac{\partial x}{\partial y}\right)_z\left(\frac{\partial y}{\partial z}\right)_u + \left(\frac{\partial x}{\partial u}\right)_z\left(\frac{\partial u}{\partial z}\right)_y = -\frac{y}{2x}\cdot\frac{z}{3y} + \frac{1}{2}\frac{u}{3x}\frac{z}{u} = \frac{1}{6}\cdot\left(-\frac{z}{x}+\frac{z}{x}\right) = 0$ ✓

b)

$\frac{1}{x}\left(\left(\frac{\partial \phi}{\partial z}\right)_u\left(\frac{\partial z}{\partial x}\right)_y + \left(\frac{\partial \phi}{\partial u}\right)_z\left(\frac{\partial u}{\partial x}\right)_y\right) + \frac{1}{y}\left(\left(\frac{\partial \phi}{\partial z}\right)_u\left(\frac{\partial z}{\partial y}\right)_x + \left(\frac{\partial \phi}{\partial u}\right)_z\left(\frac{\partial u}{\partial y}\right)_x\right) =$

$\frac{1}{x}\left[\left(\frac{\partial \phi}{\partial z}\right)_u\frac{2x}{z} + \left(\frac{\partial \phi}{\partial u}\right)_z\cdot\frac{x}{u}\right] + \frac{1}{y}\left(\left(\frac{\partial \phi}{\partial z}\right)_u\frac{y}{z} + \left(\frac{\partial \phi}{\partial u}\right)_z\cdot\frac{-y}{u}\right) =$

$\left(\frac{\partial \phi}{\partial z}\right)_u\cdot\frac{2}{z} + \left(\frac{\partial \phi}{\partial z}\right)_u\frac{1}{z} + 0 = \frac{3}{z}\left(\frac{\partial \phi}{\partial z}\right)_u$ ✓

$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$

10E

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11F

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