

2016 Mathematics (1)

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7A

Consider the mapping $z = f(\zeta)$ such that $G(z) = G(f(\zeta)) = \psi(\zeta)$, where f , G , ψ are complex functions and z , ζ are complex variables.

- (a) What condition(s) must be satisfied for $\psi(\zeta)$ to be analytic? [3]
- (b) Suppose that $\psi(\zeta) = \ln(\zeta + 2)$ and $f(\zeta)$ is defined by

$$\frac{df}{d\zeta} = \frac{i}{\sqrt{(\zeta + 1)(\zeta - 1)}}, \quad (*)$$

where $\zeta = 0$ maps to $z = 0$.

- (i) By integrating (*), show that the upper half of the ζ plane maps onto the region R defined by $|\operatorname{Re}(z)| \leq \frac{1}{2}\pi$, $\operatorname{Im}(z) \geq 0$. Determine the location of any points in the region R where $G(z)$ is not analytic. How do these relate to points in the ζ plane? [Hint: $\sin(x + iy) = \sin(x) \cosh(y) + i \cos(x) \sinh(y)$.] [7]
- (ii) The vector field $\mathbf{u} = (u, v)$ in the ζ plane is given by $u - iv = d\psi/d\zeta$. How does the magnitude of \mathbf{u} vary across the upper half of the ζ plane? In what direction is \mathbf{u} oriented? [3]
- (iii) The vector field $\mathbf{U} = (U, V)$ is defined in the region R of the z plane by $U - iV = dG/dz$. Determine this field and use a sketch to illustrate the orientation of the vector field in this region. [7]