2014 Mathematics (2)

Section A

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Section B

11S

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Solution(s):

From user: ar857

$+2^{-a}$

11 in control p_0
in tumene p_1

$P(A|D) \; \frac{A \wedge D}{P(D)}$

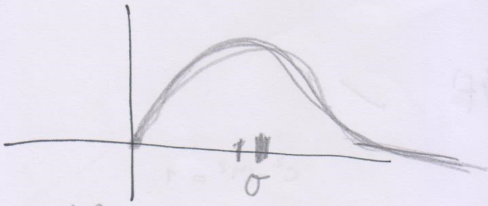
a) $p(C|E) = \frac{p(E) \wedge p(C)}{P(E)} = \frac{p_0}{p_0 + p_1}$

$-\frac{\cancel{2}x}{\cancel{2}\sigma^2}e$

b) $\cancel{A}$ $f(x) = Axe^{\frac{-x^2}{2\sigma^2}}$

$$\int_0^\infty Axe^{-\frac{x^2}{2\sigma^2}}dx = A\sigma^2 e^{-\frac{x^2}{2\sigma^2}}\Big]_0^\infty = A\sigma^2 = 1 \quad \underline{\underline{A = \frac{1}{\sigma^2}}}$$

$\lambda e^{-\lambda x}$



$\frac{x}{\sqrt{2}\sigma} = R$
$dx = \sqrt{2}\sigma dr$

$$\int_0^\infty \frac{1}{\sigma^2} x^2 e^{-\frac{x^2}{2\sigma^2}} = -x\, e^{-\frac{x^2}{2\sigma^2}}\Big]_0^\infty + \int e^{-\frac{x^2}{2\sigma^2}}\Big]_0^\infty = \sqrt{2}\,\sigma \frac{\sqrt{\pi}}{2} = \sigma\sqrt{\frac{\pi}{2}}$$

c) $\binom{m+n}{m} \theta^m (1-\sigma)^n$

wins 10 games and loozes ~~[illegible]~~ n games $n \leq 10$

$$\sum_{n=0}^{10} \binom{m+n}{n} \sigma^{10} (1-\sigma)^n \cdot \sigma$$

wins lose noes
wins last match

$$\sum_{n=0}^{10} \binom{m+n}{n} \sigma^{11} (1-\sigma)^n$$

12X

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Solution(s):

From user: ar857

2) a) i) orthogonal matrix is a matrix for which $A^TA = I$

it is always true that $AA^T = A^TA = I$

ii) $M = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ $\det M = \begin{vmatrix} 1 & 1 & -1 \\ 0 & -4 & 1 \\ 0 & 0 & 2 \end{vmatrix} = -8$

$-3-2-3=-8$ ✓

$AB = (AB)^T = B^TA^T$

$M^{-1} = -\frac{1}{8}\begin{pmatrix} -3 & -2 & -3 \\ -1 & 2 & -1 \\ 4 & -0 & -4 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 3 & 2 & 3 \\ 1 & -2 & 1 \\ -4 & 0 & 4 \end{pmatrix}$

not orthogonal

iii) $A^TA = I = AA^T$ $\quad B^TB = I = BB^T$

$A^T = A^{-1}$ $\quad B^T = B^{-1}$

$(AB)(AB)^T = ABB^TA^T = AIA^T = I$ ✓

$(AB)(AB)^T = (AB)B^TA^T = AIA^T = AA^T = I$ ✓

b) i) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & a-1 \\ a & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

ii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 2 & 1 & 4 \\ 2 & 1 & 3 & 5 \end{array}\right)$ $\quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & -1 & -2 \\ 0 & -1 & 1 & -1 \end{array}\right)$

$z=2$ $\quad y=3$

$x=3-3-2$

$x=-2$

iii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 2 & a-1 & 4 \\ a & 1 & 3 & 5 \end{array}\right)$ ~~multiple solution for~~

$\det = \cancel{6} - a + \cancel{1} - \cancel{6} + 2 + a^2 - a - 2a$

$= a^2 - 4a + 3 = (a-1)(a-3)$

no solutions for $a=3$

multiple for $a=1$

iv) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 2 & 0 & 4 \\ 1 & 1 & 3 & 5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 2 & 2 \end{array}\right) \Rightarrow \boxed{\begin{array}{l} z=1 \\ x+y=2 \end{array}}$

2014 Paper 2 12X

13W

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Solution(s):

From user: ar857

13W 2014 Exam paper 2

a) i) $\int (\cosh^2 x + \cosh x - \sinh^2 x + \sinh x)\,dx = \int (1 + \cosh x + \sinh x)\,dx = x + \sinh x + \cosh x + c$

ii) $\int \frac{x-1}{3x^2+2x+2}\,dx = \int \frac{\frac{1}{6}(6x+2)}{3x^2+2x+2} - \frac{4}{3}\int \frac{1}{3x^2+2x+2} = \frac{1}{6}\ln|3x^2+2x+2| - \frac{4}{3}\int \frac{1}{\left(\sqrt{3}x+\frac{1}{\sqrt{3}}\right)^2+\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}\,dx$

$= \frac{1}{6}\ln|3x^2+2x+2| - \frac{4}{3}\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{\arctan\left[\left(\sqrt{3}x+\frac{1}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{5}}\right]}{\sqrt{5}} + C = \frac{1}{6}\ln|3x^2+2x+2| - \frac{4\arctan\left(\frac{3x+1}{\sqrt{5}}\right)}{3\sqrt{5}} + C$

good.

b) $\int_{-1/4} \sin^2(3x^2+2x)\ln\left[\frac{1-x^5}{1+x^5}\right]dx \leftarrow$ (even function).(odd function) = odd function

$f(x)\sin^2(3x^3+2x)$ is a even ~~...~~ function

$g(x)\ln\left(\frac{1-x^5}{1+x^5}\right)$ is a odd ~~...~~ function

$g(x) = -g(-x) \Rightarrow$ odd function

$\ln(1-x^5) - \ln(1+x^5) = -\ln(1+x^5) + \ln(1-x^5)$

$\sin^2(3x^3+2x) = \sin^2(-3x^3-2x)$

$(\sin(3x^3+2x))^2 = (-\sin(3x^3+2x))^2$

$f(x) = f(-x)$

= even function

$\int_{-a}^{a}$ of a odd function $=0$

c) $\frac{d}{dx}\left[\int_a^x f(y)\,dy\right] = f(x)$

$F(x) = \int_a^x f(y)\,dy \qquad \frac{d}{dx}(Fx) = \lim_{\delta x \to 0} \frac{\int_a^{x+\delta x} f(y)\,dy - \int_a^x f(y)\,dy}{\delta x} = \lim_{\delta x \to 0} \frac{\int_x^{\delta x + x} f(y)\,dy}{\delta x} = \frac{\delta x \cdot f(x)}{\delta x} = f(x)$

nice.

d) $\frac{d}{dx}\left[\sum_{n=0}^{N}\binom{N}{n}\int_n^x \sin(y^2+y^6)\,dy\right]$

$= \sum_{n=0}^{N}\binom{N}{n}\frac{d}{dx}\int_n^x \sin(y^2+y^6)\,dy = \sum_{n=0}^{N}\binom{N}{n}\sin(x^2+x^6) = \sin(x^2+x^6)\cdot\sum_{n=0}^{N}\binom{N}{n}$

$= \sin(x^2+x^6)\cdot(1+1)^N = \sin(x^2+x^6)\cdot 2^N$

super.

$\binom{N}{0} + \binom{N}{1} + \binom{N}{2}$

14Y

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Solution(s):

From user: ar857

15T

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Solution(s):

From user: ar857

2014 II 14y

a) $df = (4x + xy^2)\,dx + (y + x^2 y)\,dy$

$$\frac{\partial P}{\partial y} = 2xy = \frac{\partial Q}{\partial x} \Rightarrow \text{is exact}$$

$$f = 2x^2 + \frac{1}{2}x^2y^2 + \frac{1}{2}y^2 + C$$

$$2x^2 + \frac{1}{2}x^2y^2 + \frac{1}{2}y^2 = D$$

$$y = \sqrt{\frac{2D - 4x^2}{x^2+1}}$$

$y(1)=2$ $\qquad 2 + \frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 4 = D$

$$y = \sqrt{\frac{12 - 4x^2}{x^2+1}}$$

c) $(3xy^2 + 2y)\,dx + (2x^2y + x)\,dy = 0$

$$\frac{6xy + 2 - 4xy - 1}{2x^2y + x} = \frac{1}{\mu}\frac{d\mu}{dx}$$

$$\frac{1}{x}\,dx = \frac{1}{\mu}\,d\mu$$

$$\mu = x$$

$$df = (3y^2x^2 + 2yx)\,dx + (2x^3y + x^2)\,dy$$

$$x^3y^2 + x^2y - C = 0$$

$$y = \frac{-x^2 \pm \sqrt{x^4 + 4Cx^3}}{2x^3}$$

b) $$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{1}{\mu}\frac{d\mu}{dx}$$

$$\ln \mu = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\,dx \Rightarrow f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

$$\exp\left(\int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\,dy\right) = \psi$$

15 T 2014 II

b) $f(x) = (1+x)^{1/3} = 1 + \frac{1}{3}x + \frac{\frac{1}{3}\cdot -\frac{2}{3}}{2}x^2 + \frac{\frac{1}{3}\cdot -\frac{2}{3}\cdot -\frac{5}{3}}{3!}x^3 \dots$

$= \sum_{n=0}^{\infty} x^n \frac{1}{n!}\frac{1}{3^n}(1 \dots (1-3(n-1))) = \sum_{n=0}^{\infty} x^n \frac{1\cdot(-2)(-5)\dots(4-3n)}{n!\,3^n}$

c) $\cos\sqrt{\frac{\pi^2}{16}+x} = \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cdot x \cdot \frac{1}{2}\frac{4}{\pi} + \frac{x^2}{2}\cdot\left(-\frac{1}{4}\frac{16}{\pi^2}\cos\frac{\pi}{4} + \frac{1}{4}\sin\frac{\pi}{4}\frac{64}{\pi^3}\right)$

$f'(x) = -\sin\sqrt{\frac{\pi^2}{16}+x}\cdot\frac{1}{2}\left(\frac{\pi^2}{16}+x\right)^{-1/2}$

$f''(x) = -\frac{1}{4}\left(\frac{\pi^2}{16}+x\right)^{-1}\cos\sqrt{\frac{\pi^2}{16}+x} - \frac{1}{2}\sin\sqrt{\frac{\pi^2}{16}+x}\cdot -\frac{1}{2}\left(\frac{\pi^2}{16}+x\right)^{-3/2}$

$\cos\sqrt{\frac{\pi^2}{16}+x} = \frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}x\frac{4}{\pi} + \frac{x^2}{2}\cdot\left(-\frac{4}{\pi^2}\cdot\frac{\sqrt{2}}{2} + \frac{16}{\pi^3}\cdot\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{\pi}x + x^2\cdot\left(-\frac{\sqrt{2}}{\pi^2} + \frac{4\sqrt{2}}{\pi^3}\right)$

d) $\ln(1+x+x^2) = \ln\left(x^2\cdot\left(\frac{1}{x^2}+\frac{1}{x}+1\right)\right) = \ln x^2 + \ln\left(\frac{1}{x^2}+\frac{1}{x}+1\right)$

$= \ln x^2 + \left(\frac{1}{x^2}+\frac{1}{x}\right) - \frac{\left(\frac{1}{x^2}+\frac{1}{x}\right)^2}{2} = \ln x^2 + \frac{1}{x} + \frac{1}{x^2} - \left(\frac{1}{x^2}\right)/2 = \ln x^2 + \frac{1}{x} + \frac{1}{2x^2}$

16R

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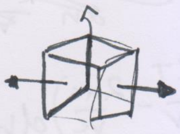
Solution(s):

From user: ar857

(16) 2014 Paper 2 16R

$V = (2xy + 2xz,\ x^2 - y^2,\ y^2 - z^2)$

i) $\nabla \cdot v = 2y + 2z - 2y - 2z = 0$



ii)

$I_1 = \int_0^1 v \cdot (-1,0,0)\, dy\, dz = \int_0^1 0\, dy\, dz = 0$

$I_2\ \int_0^1 v \cdot (1,0,0)\, dy\, dz = \int_0^1 2y + 2z\, dy\, dz$

$I_3\ \int_0^1 v \cdot (0\ 0\ 1)\, dx\, dy = \int_0^1 y^2 - 1\, dx\, dy$ $\quad (2xy + 2xz, x^2 - y^2, y^2 - z^2)$

$I_4\ \int_0^1 v \cdot (0\ 0\ -1)\, dx\, dy = \int_0^1 -y^2\, dx\, dy$

$I_5\ \int_0^1 v \cdot (0\ 1\ 0)\, dz\, dx = \int_0^1 x^2 - 1\, dx\, dz$

I_6

$= \int_0^1 -x^2\, dx\, dz$

$I = 0 + 1 + 1 - \frac{1}{3} - 1 + \frac{1}{3} + \frac{1}{3} - 1 - \frac{1}{3} = 0$

b) F_1 is $\phi = x^3 y^2 z + c$

F_2 is not

$\int_0^1 (6t^5, 4t^5, t^5) \cdot (1\ 1\ 2)\, dt = \int_0^1 12t^5 = \left[2t^6\right]_0^1 = 2$

$\int_0^1 (6t^7, 4t^7, t^5)(1\ 1\ 6t^2)\, dt\ \int_0^1 16t^7 = \left[2t^8\right]_0^1 = 2$

we expect the values to be independent of path so they should be equal

$\int_V (\nabla \cdot F)\, dV$

$\int_0^1 \int_0^1 \int_0^1 2y + 2z - 2y - 2z = 0$ ✓

check?

$\int_V (\nabla \cdot F)\, dV = \oint_S F \cdot dS$

17Z

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Solution(s):

From user: ar857

2014 Paper II 17Z

(17) $p = \frac{nRT}{V} - \frac{n^2 a}{V^2}$

a)

$$dp = \left(\frac{RT}{V} - \frac{2na}{V^2}\right)dn + \frac{nR}{V}dT + \left(2\frac{n^2 a}{V^3} - \frac{nRT}{V^2}\right)dV$$

b)

$g' \frac{1}{r}(x,y,z) = \frac{r}{|r|} g'$ $\quad r = (x,y,z)$

c) $x^2 + yz$

i) $\nabla f = (2x, z, y)$

$(2,-3,1)\cdot\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right) = \frac{4}{3} - 1 + \frac{2}{3} = 1$

ii) $\frac{(2,-3,1)}{\sqrt{14}}$

18T

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19R*

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20Z*

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